Task Decomposition Through Competition
in a Modular Connectionist Architecture:
The What and Where Vision Tasks

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A novel modular connectionist architecture is presented in which the networks
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decomposition in the sense that it learns to partition a task into two or more functionally
independent tasks and allocates distinct networks to learn each task.
In addition, the architecture tends to allocate to each task the network whose
topology is most appropriate to that task. The architecture's performance on
"what" and "where" vision tasks is presented and compared with the performance
of two multilayer networks. Finally, it is noted that function decomposition is an underconstrained problem, and, thus, different modular architectures
may decompose a function in different ways. A desirable decomposition can be
achieved if the architecture is suitably restricted in the types of functions that it
can compute. Finding appropriate restrictions is possible through the application
of domain knowledge. A strength of the modular architecture is that its structure
is well suited for incorporating domain knowledge.

Although many biologists and psychologists share the view that the brain has a modular architecture, there is no general agreement on the number of
modules, the function of the modules, the nature of the interaction between
modules, or the manner in which the modules develop. One reason for this

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diversity of opinion is that answering questions about the modular nature of the brain involves the difficult task of reasoning about a system with a large number of interacting components. Even systems of interacting components with a small fraction of the brain's complexity present formidable conceptual and analytical difficulties. In many cases, mathematical and computer models provide essential tools for understanding aspects of these systems. One class of models that has the potential for helping to answer questions about modular systems is the class of connectionist models, also known as artificial neural network models.

A hierarchical classification of the components of connectionist models may be defined in which a unit is the finest level of classification, a layer is a coarser level, and a network is a still coarser level. Connectionist researchers typically design systems that are modular at the level of units or layers. In this article we argue that there are significant practical and theoretical advantages to be realized by considering modularity at the level of networks. In other words, we argue that connectionist architectures should consist of multiple networks, and that connectionist learning algorithms should be designed to take advantage of this modular structure.

Although terms such as layer or network are imprecise, it is generally agreed that they provide a convenient language for discussing connectionist architectures. An analogous situation occurs in the neurosciences where researchers debate whether nervous systems are best conceptualized at the level of the neuron, or at coarser levels such as the column, hypercolumn, or area (Kaas, 1982). This debate persists despite the lack of agreement on the precise definitions of the coarser taxonomic levels. For our purposes, we rely on the intuition that portions of an architecture larger than a unit or layer, which learn and perform different functions, constitute different networks.

This article introduces a novel modular connectionist architecture in which the networks composing the architecture compete to learn the training patterns. An outcome of this competition is that different networks learn different training patterns and, thus, learn to compute different functions. The architecture performs task decomposition in the sense that it learns to partition a task into two or more functionally independent tasks and allocates distinct networks to learn each task. In addition, it tends to allocate to each task the network whose topology is most appropriate to that task. This property has implications for both the efficiency of learning and the forms of generalization the modular architecture produces.

The article is organized as follows. Section 1 discusses computational advantages of modular connectionist architectures that are capable of allocating different networks to compute different functions. Section 2 describes the modular architecture that we have developed. Section 3 describes the "what" and "where" vision tasks studied by Rueckl, Cave, and Kosslyn (1989). Section 4 compares the performance of our modular architecture with that of two networks using the backpropagation algorithm (LeCun, 1985; Parker, 1985; Rumelhart, Hinton, & Williams, 1986; Werbos, 1974) on the what and where vision tasks. Section 5 discusses the relationship between the structure of the modular architecture and the nature of the task decomposition discovered by the architecture.

1. ADVANTAGES OF MODULAR CONNECTIONIST ARCHITECTURES

Theorists have shown that connectionist networks are universal approximators, meaning that, for any given function, there is a connectionist network capable of approximating it arbitrarily closely (e.g., Hornik, Stinchcombe, & White, 1989). Given this result, what advantages might a modular architecture consisting of several connectionist networks have over a single network? In this section we answer this question by arguing that modular architectures have advantages in terms of learning speed, generalization capabilities, representation capabilities, and their ability to satisfy constraints imposed by hardware limitations. This discussion is not specific to the architecture that we have developed; it applies to the general class of multinet network systems of which ours is an example.

1.1 Learning Speed

Several characteristics of modular architectures suggest that they should learn faster than single connectionist networks. One such characteristic is that modular architectures can take advantage of function decomposition. If there is a natural way to decompose a complex function into a set of simpler functions, then a modular architecture should be able to learn the set of simpler functions faster than a single network can learn the undecomposed complex function. For example, consider the absolute value function

\[ f(x) = \begin{cases} \ -x & \text{if } x < 0 \\ \ x & \text{if } x \geq 0 \end{cases} \]

This nonlinear function can be learned by a single network with at least one layer of hidden units. Alternatively, it can be learned by a modular architecture consisting of two networks, each a single linear unit, and a mechanism for switching on the appropriate network in the appropriate context. One network can learn the function \( f(x) = x \), and the other network can learn the function \( f(x) = -x \). Assuming that it is relatively easy to learn the switching mechanism, the modular architecture should be able to learn faster than the single network because it does not use hidden units and each module only needs to learn a linear function.

In addition to being able to take advantage of function decomposition, modular architectures can be designed to reduce the presence of conflicting
training information that tends to retard learning. We refer to conflicts in training information as cross-talk and distinguish between spatial and temporal crosstalk. Spatial crosstalk occurs when the output units of a network provide conflicting error information to a hidden unit. Jordan (1986) and Plaut and Hinton (1987) noted that this occurs when the backpropagation algorithm is applied to a single network containing a hidden unit that projects to two or more output units. For example, suppose that a hidden unit projects via positive weights to two output units and that, when compared to the desired output values, the output level of the first output unit is too small, whereas the output level of the second output unit is too large. Using the backpropagation algorithm, the first output unit provides derivative information specifying that the hidden unit should have a larger output. However, the second output unit provides derivative information specifying that this same hidden unit should have a smaller output. This conflict in derivative information is an instance of spatial crosstalk.¹

Plaut and Hinton (1987) noted that a modular architecture consisting of a separate network for each output unit is immune to spatial crosstalk. For example, consider the systems shown in Figure 1. Panel A shows a single network and Panel B shows a modular architecture consisting of three separate networks, one for each output unit. Although these systems can be applied to the same tasks, the modular architecture is immune to spatial crosstalk because each hidden unit projects to a single output unit.

In contrast to spatial crosstalk—where a unit receives inconsistent training information at a single instant in time—a unit might receive inconsistent training information at different times, a situation resulting in temporal crosstalk. One form of temporal crosstalk occurs when a network is trained to perform different functions at different times. For example, suppose that when a network is trained to perform one function, some of its hidden units become particularly useful in performing that function. When this same network is later trained to perform a second, different function, one would like it to learn the second function without unnecessarily degrading its performance on the first function. This can be accomplished if the hidden units that participate in implementing the second function are not the ones that implement the first function. However, as Sutton (1986) observed, gradient descent algorithms produce the opposite behavior because they preferentially modify the weights of the hidden units that are already useful. Therefore, after training on the second function, the network’s performance on the first function will tend to be significantly degraded.

¹ Although spatial crosstalk is clearly seen in terms of the backpropagation algorithm, it is not limited to networks trained using this algorithm. A network trained using any algorithm that approximates gradient descent (e.g., the Aδξ−p algorithm of Barto & Anandan, 1985; Barto & Jordan, 1987) can suffer from spatial crosstalk.

Temporal crosstalk arises in another situation closely related to that just described: when training patterns come from one region of the input space for many consecutive trials, and later come from a different region for many consecutive trials.² If the function has different properties in these different regions, the network would receive conflicting training information. Training with patterns in the second region of the input space can result in degradation of the network’s ability to perform correctly for patterns in the first region.

The conflicts in training information producing spatial and temporal crosstalk can be thought of in terms of transfer of training. Training a system to compute one function may facilitate the system’s ability to learn a second function, a situation referred to as positive transfer of training, or it may retard the system’s ability to learn a second function, a situation referred to as negative transfer of training. Suitably designed modular architectures should learn faster than single networks because similar functions can be learned by the same network of the modular architecture, resulting in the benefits of positive transfer of training, whereas dissimilar functions can be learned by different networks, thereby avoiding the detrimental effects of negative transfer of training.

1.2 Generalization
Unless the structure of a network is well matched to the function on which it is trained, it is unreasonable to expect the network to generalize well (e.g., Denker et al., 1987). For example, although both a single-layer linear net-

² Although this is not how one would typically prefer to train a network, it may be unavoidable. For example, in training a network to control a dynamical system, the state of the dynamical system is likely to change slowly relative to the rate at which the system’s variables are observed. Hence, the network is likely to receive inputs from the same region of the input space for many consecutive time steps.
work and a multilayer nonlinear network can accurately learn a set of training pairs generated by a linear function, one would expect the single-layer network to generalize better because its structure is closer to the structure of the function being learned. If a modular architecture were able to decompose a complex function into a set of simpler functions and allocate an appropriately structured network to each simpler function, then one would expect good generalization. In this case, the mechanism responsible for allocating patterns to the networks of the modular architecture would be automating part of the process of matching network structures to tasks, the other part of the process being played by the initial choice of the modular architecture's repertoire of networks.

A second reason that a modular architecture should generalize better than a single network involves the difference between local and global generalization. Global generalization occurs when the learning of a training pattern from one region of the input space influences the network's performance on patterns from a much wider region of the input space. As previously suggested in the discussion of temporal cross-talk, when the function generating the training patterns possesses different characteristics in different regions, global generalization is an undesirable property due to negative transfer of training. In contrast, modular architectures perform local generalization in the sense that each network of the architecture only learns patterns from a limited region of the input space. Therefore, training a modular architecture on a training pattern from one of these regions does not affect the architecture's performance on patterns from the other regions.

1.3 Representation

Modular architectures tend to develop representations that are more easily interpreted than the representations developed by single networks. By this we mean that it tends to be easier to understand how a modular architecture implements a function than it is to understand how a single network implements the same function. This property was demonstrated by Rueckl et al. (1989) who trained two connectionist systems to perform object recognition and spatial localization from simulated retinal images. As a result of learning, the hidden units of the system using separate networks for the recognition and localization tasks contribute to the solutions of these tasks in more understandable ways than the hidden units of the single network applied to both tasks. In the former system, a different set of hidden units is used to represent information about the different tasks. In the latter system, on the other hand, the same set of hidden units is forced to represent information about both tasks despite the fact that the two tasks are relatively independent.

In addition to making it easier for experimenters to understand their systems, interpretable representations may make it easier for one portion of a system to understand the tasks performed by other portions. Minsky (1986) argued that if one module can determine the task that is performed by a second module, then the former module can request the latter module to perform that task in the appropriate situation. This view emphasizes that the functions learned by the modules can be thought of as "building blocks" to be used on other occasions in the performance of more complex tasks.

Another advantage of using different networks to perform different portions of a task is that this representation facilitates the use of attentional mechanisms. Cowey (1985) speculated that properties of a visual image may be attended to or unattended to through the operation of a relatively coarse mechanism that enhances or suppresses the activations of entire populations of processing units. Only if different modules of an architecture represent different portions of a task would we generally expect such mechanisms to have coherent and sensible effects.

Other reasons that modular architectures are superior to single networks in their representational capabilities are related to familiar arguments advanced in the computer science and psychology literatures: Because modular architectures localize functions and develop more interpretable representations, they are easier for researchers to debug; because much of human knowledge is modularized, it is easier for researchers to embed domain knowledge into a connectionist system when the system is organized in a modular fashion; and because modules can be added one at a time, modular architectures facilitate the development of connectionist systems in an incremental manner.

1.4 Hardware Constraints

Another property of modular architectures suggesting advantages over single networks is that modular architectures can more closely satisfy several types of constraints imposed at the level of hardware implementation. In particular, suitably designed modular architectures can reduce the number of units and the lengths of connections.

In a discussion of representations employed by the brain, Ballard (1986) suggested that a limitation on the number of neurons compels the brain to adopt a modular architecture. He hypothesized that the brain uses a coarse code (Albus, 1975; Hinton, 1981b) to represent multidimensional spaces. Using this kind of representation, the number of neurons required to represent a space is $N^k/D^{k-1}$, where $k$ is the dimension of the space, $N$ is the number of just-noticeable differences in each dimension, and $D$ is the diameter of the receptive field of each neuron (Ballard, 1986; Hinton, 1981b). Because this rapid growth in the number of neurons limits the number of dimensions that can be represented in a cortical area, high-dimensional spaces must be represented in such a way that different dimensions are represented in different areas. Because different areas represent different dimensions, these areas must compute different functions. Analogously, in order
to reduce the number of units required by a connectionist system, one may
distribute the representation of multidimensional spaces among multiple
networks.

Other speculations as to why the brain contains multiple cortical areas
are also relevant to connectionist systems. Cowey (1981) suggested that if
the cortex uses lateral inhibition to sharpen various visual attributes, such as
dges, orientation, color, disparity, spatial frequency, size, and movement,
then a retinotopic representation of these attributes allows the use of rela-
tively short connections between neurons because this representation places
the interneurons necessary for receptive-field tuning relatively close together
(Durbin & Mitchison, 1990; Kohonen, 1982). If neurons for all the different
attributes are represented in a single retinotopic map, however, the connec-
tions needed to sharpen the tuning of individual neurons for one of these
attributes would be unnecessarily long. The existence of multiple retinotopic
maps, each in a separate cortical area, allows the cortex to highlight several
attributes using short local connections.

Similarly, Barlow (1986) suggested that, in order to detect various visual
attributes using neurons whose connections are of minimal length, the cor-
tex must employ multiple representations. He argued that because the brain
primarily contains local connections, it is difficult, using a retinotopic rep-resenta-
tion, to detect similarities among noncontiguous locations of the visual
field. In order to detect such similarities, it is necessary to map the
information in the retinal image so that similar events are represented close
to each other independently of the retinal coordinates of the events. For
example, using a retinotopic representation and neurons with local connections,
it is difficult to detect the collinearity of line segments located at different
places in the retinal image. However, in a different, nonretinotopic repre-
sentation (e.g., the Hough transform, Ballard, 1984; Duda & Hart, 1973),
nearly collinear line segments can be represented by neighboring neurons. In
general, the existence of multiple representations, each in a separate area,
allows the brain to detect several attributes using short connections.

The hypotheses of Ballard (1986), Cowey (1981), and Barlow (1986) about
why the cortex employs multiple representations located in different cortical
areas, are also relevant to connectionist architectures. These hypotheses
suggest that, in order to evaluate several attributes using units with local
connections, connectionist systems should employ multiple representations
located in different networks.

2. MODULAR CONNECTIONIST ARCHITECTURE

In this section we introduce a modular connectionist architecture that learns
to partition a task into two or more functionally independent tasks and allo-
cates distinct networks to learn each task.

2.1 Output of the Architecture

The architecture illustrated in Figure 2 consists of two types of networks: 
expert networks and a gating network. The expert networks compete to
learn the training patterns, and the gating network mediates this competi-
tion. After training, Expert Networks 1 and 2 compute different functions
that are useful in different regions of the input space. Let the vectors $y_1$
and $y_2$ denote the outputs of the two expert networks. The gating network is an
administrative agency that decides whether Expert Network 1 or 2 is cur-
cently applicable. Let the scalars $g_1$ and $g_2$ denote the two output units of
the gating network. The output of the entire architecture, $y$, is $g_1y_1 + g_2y_2$.

Therefore, when $g_1 = 1$, and $g_2 = 0$, Expert Network 1 determines the output
of the architecture, whereas when $g_1 = 0$, and $g_2 = 1$, Expert Network 2 deter-
mines the output. In general, the architecture may contain any number of
expert networks. If there are $n$ expert networks, then the gating network has
$n$ output units, and the architecture's output is

$$y = \sum_{i=1}^{n} g_i y_i$$

(2)

2.2 Training of the Architecture

During training, the weights of all networks are modified simultaneously
using the backpropagation algorithm. However, the learning rules used to
train the expert and gating networks are based on the minimization of dif-
ferent error functions. At each time step, the weights of the expert networks
are modified so as to reduce the sum of squared error between the output of
the system, $y$, and the desired output, $y^*$. This error, denoted $J_y$, is given by

$$J_y = \frac{1}{2} (y^* - y)^T(y^* - y).$$

(3)

The weights of the gating network are modified so as to reduce a more com-
plicated error function. The intuition behind this function is as follows. For
each training pattern, one expert network comes closer to producing the desired output than the other expert networks. In the competition among networks, this one is called the "winner" and all others are called "losers." If, on a given training pattern, the system's performance is significantly better than it has been in the past, then the weights of the gating network are adjusted to make the output corresponding to the winning expert network increase towards 1 and the outputs corresponding to the losing expert networks decrease towards 0. Alternatively, if the system's performance has not improved, then the gating network's weights are adjusted to move all of its outputs towards some neutral value.

This intuition is expressed mathematically as follows. First, it is necessary to specify what it means for the system's performance to be significantly better than it was in the past. If \( t \) is the current time step, then the error \( J_f(t) \) is a measure of the current performance. We measure the system's past performance by forming an exponentially weighted average of \( J_f \) over time steps earlier than \( t \). This value, denoted \( \bar{J}_f \), is computed iteratively by the following difference equation:

\[
\bar{J}_f(t) = \alpha J_f(t) + (1 - \alpha) \bar{J}_f(t - 1),
\]

where \( \alpha, 0 \leq \alpha < 1 \), determines how rapidly past values of \( J_f \) are forgotten. We use binary variables \( \lambda WTA \) (\( WTA \) stands for "winner-take-all") and \( \lambda NT \) (\( NT \) stands for "neutral") to indicate whether the system's performance has significantly improved. Specifically,

\[
\begin{align*}
\text{If} & \quad J_f(t) < \gamma \bar{J}_f(t - 1), \\
\text{Then} & \quad \lambda WTA = 1 \quad \text{and} \quad \lambda NT = 0 \\
\text{Else} & \quad \lambda WTA = 0 \quad \text{and} \quad \lambda NT = 1,
\end{align*}
\]

where \( \gamma \) is a multiplicative factor that determines how much less the current error must be than the measure of past errors in order for the system's performance to be considered significantly improved.

If the architecture's performance has significantly improved (\( \lambda WTA = 1 \)), we determine which expert network's output is closest to the desired output. Define the error for expert network \( i \) to be the sum of squared error between the expert network's output, \( y_i \), and the desired output, \( y^* \). This value, denoted \( J_{ni} \), is

\[
J_{ni} = \frac{1}{2} (y^* - y_i)^T (y^* - y_i).
\]

The winning expert network is the network with the smallest error. If expert network \( i \) is the winner, then the desired value of the \( i \)th output unit of the gating network, denoted \( g_{i}^* \), is set to 1. Otherwise, if expert network \( i \) is a loser, \( g_{i}^* \) is set to 0. If the architecture's performance has not significantly improved (\( \lambda NT = 1 \)), then the weights of the gating network are adjusted so that all the outputs of the gating network are moved towards a neutral value. This value is \( 1/n \), where \( n \) is the number of expert networks.

Using the quantities just defined, it is possible to write the gating network's error function, \( J_G \), as:

\[
J_G = \lambda WTA \left[ \frac{1}{2} \sum_{i=1}^{n} (g_i^* - g_i)^2 \right] + \lambda NT \left[ \frac{1}{2} \sum_{i=1}^{n} g_i (1 - g_i) \right] + \lambda NT \left[ \frac{1}{2} \sum_{i=1}^{n} \frac{1}{n} - g_i \right].
\]

Due to the definition of \( \lambda WTA \) and \( \lambda NT \) (Equation 5), the first three terms of Equation 7 contribute to the error when the architecture's performance has significantly improved, whereas only the fourth term contributes to the error when the system's performance has not significantly improved. The first term is the sum of squared error between the desired outputs and the actual outputs of the gating network. The second term takes its smallest value when the outputs of the gating network sum to 1. The third term takes its smallest value when the outputs of the gating network are binary valued. Therefore, the effect of changing the gating network's weights to reduce the second and third terms is that, in response to each input pattern, one output unit approaches 1 and all others approach 0. The fourth term is the sum of squared error between the neutral value and the actual outputs of the gating network. Reducing this term, which occurs only when the architecture's performance has not significantly improved, causes all of the outputs of the gating network to approach the neutral value \( 1/n \).

### 2.3 Discussion

These equations imply that there are three types of interactions among the networks of the modular architecture. The first type of interaction is that the gating network determines how much each expert network contributes to the output of the system (Equation 2); the second is that the performances of the expert networks determine the desired outputs of the gating network; and the third is that the gating network determines how much each expert network learns about each training pattern.

Referring to Figure 2, the error vector backpropagated into Expert Network 1 is \( g_1 (y^* - y) \), and the error vector backpropagated into Expert Network 2 is \( g_2 (y^* - y) \). This means that, in addition to determining how much each expert network contributes to the output of the architecture, the gating
network also determines the magnitudes of the expert networks’ error vectors, and therefore determines how much each expert network learns about each training pattern. This interaction between the expert networks and the gating network implements a kind of credit assignment policy whose ramifications can be clarified by the following two examples.

Suppose that the gating network responds to an input pattern with \( g_1 = 1 \) and \( g_2 = 0 \). This implies that the output of the architecture, \( y \), is the output of Expert Network 1. In this case, the error vector backpropagated to Expert Network 1 is \( y^* - y \), and the error vector backpropagated to Expert Network 2 is the zero vector. Thus, the first expert network is the one that learns about the function that generated the current training pattern, and the second expert network does not adjust its weights at all. This assignment of credit is logical because Expert Network 1 is solely responsible for the output of the architecture.

As a second example, suppose that during training of the architecture, Expert Network 1 is more closely approximating a given training pattern than is Expert Network 2, and \( g_1 \) is slightly larger than \( g_2 \). In this case, the first expert network receives a larger error and learns more about the function that generated the training pattern than the second expert network. Consequently, Expert Network 1 learns to perform this function even better than Expert Network 2, which causes \( g_1 \) to grow even larger than \( g_2 \). Thus, this credit-assignment policy produces a positive feedback effect in the sense that it enhances the performance advantage of the expert network that is already most closely approximating the current target vector.

This credit-assignment policy causes the modular architecture to allocate different expert networks to different tasks. Due to the positive feedback effect, one expert network learns the training patterns that compose a task. However, when later presented with the patterns that compose a second task, the network that won the competition to learn the patterns from the first task is unlikely also to win the competition to learn the new training patterns (unless the two tasks are very similar). Therefore, a different expert network wins the competition to learn the new training patterns. A consequence of the competition to learn the training patterns is that different expert networks learn to perform different tasks.

2.4 Relationship to Previous Research

**Competitive Learning.** A common feature of our modular architecture and some systems previously proposed is the use of competitive learning. There are many unsupervised learning algorithms that make use of competition between the units of a network (e.g., Durbin & Willshaw, 1987; Grossberg, 1987; Kohonen, 1982; Reggia, 1987; Rumelhart & Zipser, 1986; Yuille & Grzywacz, 1989). In all of these systems, the units of a single network compete for the right to respond maximally to a given subset of input patterns. The competition is based on the relative amount of match between the input vector and each weight vector of the network. As a result of this competition, different units learn to respond to patterns from different portions of the input space.

A distinguishing feature of the competitive aspects of the architecture presented here is that the objective is not to cluster the input patterns into natural groupings, but rather to cluster the training patterns, which are input patterns together with desired output patterns, into natural groupings. The competition is not between different units of a network, but rather is between different networks of the modular architecture. The expert networks compete for the right to learn a given subset of training patterns. As a result of this competition, different expert networks learn different training patterns and, thus, learn to compute different functions. A second distinguishing feature of the architecture is that the competitive process is supervised; the competition is based on the abilities of the expert networks to approximate the desired output values.\(^3\)

**Multiplicative Connections.** A second feature our modular architecture has in common with systems previously proposed is its use of multiplicative connections. Many investigators have incorporated multiplicative connections in the design of their system so that the system can compute different functions in different contexts (e.g., Feldman, 1982; Feldman & Ballard, 1982; Hinton, 1981a; Maxwell, Giles, Lee, & Chen, 1986; McClelland, 1986; Pomerleau, 1987; Sejnowski, 1981; Yeung & Bekey, 1989). Most relevant to our use of multiplicative connections is the work of Hampshire and Waibel (1989) and Pollack (1987).

Independently of the research described here, Hampshire and Waibel (1989) developed an architecture similar to the one presented in Figure 2. They refer to this architecture as a “meta-pi” network. However, their system does not perform task decomposition. It is trained in two stages. In the first stage, a useful task decomposition is provided to the system, and each expert network is separately trained to perform one of the subtasks. In the second stage, the gating network is trained to switch in the appropriate expert network in the appropriate context. Because the meta-pi network does not itself perform task decomposition, the issues studied by Hampshire and Waibel are different from the issues we are addressing.

Pollack (1987) developed a system, called the cascaded backpropagation (CBP) architecture, that uses multiplicative connections in order to learn to compute different functions in different contexts. Pollack has shown that...

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\(^3\) Further developments relating the class of modular architectures proposed here, and competitive learning can be found in Jacobs, Jordan, Nowlan, & Hinton (1991).
the CBP architecture is capable of decomposing a task into a set of subtasks, and of learning the set of subtasks faster than a single network can learn the undecomposed task. Disadvantages of this system are that the number of units required by the system scales poorly with the size of the task, and the system uses the same network topology to perform all tasks.

**Stochastic Learning Automata.** The gating network uses a learning procedure related to stochastic learning automata (Narendra & Thathachar, 1989). Stochastic learning automata maintain a probability distribution over a set of actions. At each time step, an action is selected according to this distribution. If the environment provides the automaton with a reward, then the probability of performing the selected action is increased, whereas, if the environment provides the automaton with a penalty, the probability of performing the selected action is decreased. After modifying the probability of performing the selected action, all the action probabilities are modified so that they sum to 1. The learning procedure of the gating network is similar to this, where the expert networks correspond to actions, and the outputs of the gating network correspond to action probabilities. According to this view, if the performance of the system has improved ($J_f(t) < \gamma J_p$), then the gating network is rewarded; otherwise, it is penalized.

**Brain Lateralization.** The idea of competition between networks has appeared in the cognitive neuroscience literature in the form of the hypothesis that hemispheric specialization in humans is due to competition between neural subsystems. For example, Kosslyn (1987) proposed that the brain contains many processing subsystems, each a neural network, which compete to learn about inputs. If the output of a network is used in subsequent computational processing, then the weights among connections in that network are altered so that the network produces the output faster and with less noise when the input recurs in the future. The weights of the networks whose outputs were not used in subsequent processing remain unchanged. Consequently, the networks compete to have their outputs used, and the strength of the training information received by a network is directly related to how that network fares in the competition. Kosslyn’s credit-assignment policy is thus nearly identical to the credit-assignment policy of the modular architecture previously described, and his hypothesis about why different subsystems of the brain learn to perform different tasks is consistent with our reasoning about why different expert networks of the modular architecture learn to perform different tasks.

Theories of brain lateralization also include the hypothesis that asymmetries in the cerebral hemispheres may influence the lateralization of brain functions (Geschwind & Galaburda, 1987). For example, if the left and right hemispheres compete for the ability to process language, then anatomical differences between the two hemispheres may bias the competition so that the left hemisphere usually wins. Similarly, a property of the modular architecture presented here is that the expert networks’ architectures influence the competition between these networks. The architecture tends to allocate to each task the expert network whose topology is most appropriate for that task. For example, when required to perform a linear task and a nonlinear task, a linear expert network tends to win the competition to learn the linear task, whereas a nonlinear network tends to win the competition to learn the nonlinear task. A demonstration of this property is provided in Section 4 where we compare the performances of the modular architecture with that of two other networks trained with the backpropagation algorithm on the "what" and "where" vision tasks studied by Rueckl et al. (1989).

### 3. THE "WHAT" AND "WHERE" VISION TASKS

Despite the fact that a variety of images are produced on the retina of a person watching a rotating or translating object, people recognize that the same object is depicted in each of the images. The ability to perform object recognition is said to be orientation and translation invariant. One hypothesis about how this invariance is achieved is that a canonical representation of each familiar object is stored, and the retinal image of an object is transformed so that the image and the representations can be compared. As a result of this transformation, information relevant to determining an object’s spatial location is lost. This suggests that the process performing object recognition does not also perform spatial localization. Neuroscientists speculate that distinct cortical pathways of the primate visual system compute object recognition and spatial localization. Mishkin, Ungerleider, and Macko (1983) reviewed evidence that a pathway running ventrally, interconnecting the striate, prestriate, and inferior temporal areas, computes object recognition, whereas a pathway running dorsally, interconnecting the striate, prestriate and inferior parietal areas, computes spatial localization.

To investigate the computational advantages of employing distinct systems to perform these two tasks, Rueckl et al. (1989) compared the performance of two connectionist systems on an object-recognition task (henceforth referred to as the "what" task) and a spatial localization task (henceforth referred to as the "where" task). The first system consisted of a single network that was required to perform both tasks. The second system consisted of two networks, one for each task. The retina was represented as a $5 \times 5$ binary matrix. Each object was a specific pattern of binary entries in a $3 \times 3$ matrix. At each time step of the training period, one of nine object matrices was centered at one of nine locations on the retinal matrix. The entries of the retinal matrix that lie outside the object matrix were set to 0. The what task is to identify the object; the where task is to identify its location.
Table 1: Systems Studied by Rueckl, Cave, and Kosslyn (1989) on the "What" and "Where" Tasks

<table>
<thead>
<tr>
<th>System</th>
<th>Networks</th>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25→18→18</td>
<td>retinal matrix</td>
<td>what and where</td>
</tr>
<tr>
<td>2</td>
<td>25→14→9</td>
<td>retinal matrix</td>
<td>what</td>
</tr>
<tr>
<td></td>
<td>25→4→9</td>
<td>retinal matrix</td>
<td>where</td>
</tr>
</tbody>
</table>

The single-network system that Rueckl et al. (1989) applied to these tasks was a network with two layers of modifiable weights. It had 25 input units, 18 hidden units, and 18 output units (see Table 1) and was strictly layered, meaning that all input units were connected to all hidden units, which in turn were connected to all output units. The input units encoded the 5×5 matrix, and the 18 output units corresponded to the nine possible objects and nine possible locations. The second system studied by Rueckl et al. (1989) was a modification of this single-network system. Whereas the single-network system had connections from all hidden units to all output units, the second system only had connections from the first 14 hidden units to the first 9 output units and from the remaining 4 hidden units to the remaining 9 output units (see Table 1). Thus, this system consisted of two separate networks, one for the what task, and the other for the where task. Both the single-network system and the two-network system learned by means of the backpropagation algorithm.

Simulations showed that the two-network system learned the tasks faster and developed more interpretable representations than the single-network system. According to Rueckl et al. (1989), both of these advantages resulted from the fact that the hidden units of the single-network system received inconsistent training information because they were connected to the output units for both the what and where tasks. Thus, the single-network system suffered from spatial crosstalk. In contrast, the hidden units of the two-network system did not receive inconsistent training information because they were connected to the output units for only one task. Based on these results, Rueckl et al. (1989) concluded that it is better for a connectionist system—and, by analogy, the primate visual system—to perform the what and where tasks in distinct networks.

An issue that Rueckl et al. (1989) did not address, and the issue with which we are primarily concerned, is the development of a system that can learn for itself if it is better to decompose a learning task into two or more simpler tasks, and if so, can allocate distinct networks for learning each simpler task. The next section presents simulation experiments using the what and where tasks to demonstrate that our modular architecture has this ability.

4. Simulation Experiments

Simulations of our modular architecture applied to the what and where tasks were conducted to investigate the architecture's ability to decompose learning tasks and the advantages that this ability may provide. For comparative purposes, several single networks were also simulated as applied to these tasks. Two sets of simulations were conducted. One set was designed to examine the modular architecture's behavior in the presence of temporal crosstalk; the other set of simulations addresses spatial crosstalk.

In describing the simulations, we refer to three different time periods: at each time step, a system is presented with a single input–output pair; during each epoch, a system is presented with every input–output pair in the training set exactly once; and a run consists of 100 epochs. In all simulations, the measure of a system's performance is the percentage of input–output pairs that the system performs correctly during each epoch. A training pair is considered to be performed correctly when each output unit of the system has an activation greater than .6 when its desired activation is 1 and an activation less than .4 when its desired activation is 0. The results for each system were averaged over 25 runs. For many parameters of each system (e.g., step size and momentum), we used the values that appeared to give the best performance. These values and additional details of the simulations are provided in the Appendix.

4.1 Temporal Crosstalk

Recall that temporal crosstalk occurs when a system is trained to perform different tasks at different times. We trained a modular architecture and single networks to perform that what and where tasks. At any given time step, each system was trained to perform only one task. In addition to receiving 25 input values corresponding to the entries of the 5×5 retinal matrix, each system received an input indicating whether it should perform the what or the where task. We call this input the task specifier. There are 162 different input–output pairs (9 objects × 9 locations × 2 tasks).

Two training procedures were employed to test each system's robustness in the presence of temporal crosstalk. One procedure avoids temporal crosstalk whereas the other does not. In the training procedure that avoids temporal crosstalk, called random training, at each time step an input pattern, consisting of a retinal matrix and the task specifier, is randomly selected according to a uniform distribution over the set of 162 possible input patterns.

* As noted by Rueckl et al. (1989), the what and where tasks are crude analogues, not realistic correlates, of problems confronted by biological visual systems. Furthermore, it is unlikely that the brain learns to use the temporal areas for object recognition and the parietal areas for spatial localization. As used by Rueckl et al. and in this article, the what and where tasks are intended merely to give a concrete context for examining abstract issues.

* The more common sum of squared error measure was also computed and yielded results qualitatively similar to those produced using the percent correct. Thus, the sum of squared error is not reported here.
TABLE 2A
Systems Studied in Temporal Crosstalk Experiments: Single Networks

<table>
<thead>
<tr>
<th>System</th>
<th>Networks</th>
<th>Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26→18→9</td>
<td>retinal matrix and task specifier</td>
</tr>
<tr>
<td>2</td>
<td>26→36→9</td>
<td>retinal matrix and task specifier</td>
</tr>
</tbody>
</table>

TABLE 2B
Systems Studied in Temporal Crosstalk Experiments: Modular Architecture

<table>
<thead>
<tr>
<th>System</th>
<th>Expert Networks</th>
<th>Expert Networks' Input</th>
<th>Gating Network</th>
<th>Gating Network's Input</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>26→36→9</td>
<td>retinal matrix and task specifier</td>
<td>1→3</td>
<td>task specifier</td>
</tr>
<tr>
<td></td>
<td>26→18→9</td>
<td>retinal matrix and task specifier</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>26→9</td>
<td>retinal matrix and task specifier</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
There are at least three ways that this modular architecture might successfully learn the what and where tasks. One of the multilayer expert networks could learn to perform both tasks. In this case, the gating network must always gate on the network that learns both tasks and gate off the remaining expert networks. Although this is a possible solution, the previously described results of Rueckl et al. (1989) suggest that it is not the best solution in terms of learning speed and clarity of the resulting representation. A second possibility is that one of the multilayer expert networks could learn the what task, and the other multilayer expert network could learn the where task. In this case, the gating network must gate on the appropriate expert network based on the value of the task specifier. This solution would indicate that the modular architecture had learned that it was required to perform two independent tasks and had allocated distinct networks to each task. However, a shortcoming of this solution is apparent when it is noted that, using the retinal images designed by Rueckl et al., the where task is linearly separable. This means that the structure of the single-layer expert network most closely matches the where task. Consequently, a third and possibly best solution would be one in which one of the multilayer expert networks learned the what task and the single-layer expert network learned the where task. This solution would not only show task decomposition, but also the appropriate allocation of tasks to expert networks.

The simulation experiments using the modular architecture show that it produces this third possible solution. It always allocates the first multilayer expert network to the what task and the single-layer expert network to the where task. This result suggests that, at least in some circumstances, the modular architecture is capable of performing function decomposition and that it tends to allocate to each function a network with a structure appropriate to that function. Learning curves for the modular architecture on the what and where tasks using random and blocked training are shown in Figure 6 (p. 240). Because little different exists between performance with random and blocked training, these results suggest that the modular architecture is robust in the presence of temporal crosstalk (at Epoch 50, the difference between the performance with random and blocked training is not statistically significant at the $p < .01$ level). This robustness is due to the architecture's ability to allocate distinct networks to learn the different tasks.

A comparison of the learning curves in Figure 3, 4, and 6 shows that both the 26–36–9 network and the modular architecture learn the what and where tasks faster than the 26–18–9 network. In addition, the results suggest that of the three systems studied, only the modular architecture is capable of showing robust performance in the presence of temporal crosstalk.

4.2 Spatial Crosstalk
As presented so far, a limitation of the modular architecture is that only one expert network determines the output of the architecture at any one time.
Consider the architecture illustrated in Figure 7. It consists of two expert networks and two gating networks. One gating network gates the first $m$ components of each expert network’s output vector, and the other gating network gates the remaining components. Suppose one function determines the desired values for the first $m$ components of the architecture’s output vector, $y$, and a second function determines the desired values for the remaining components. The gating networks may allocate the same expert network to both functions, or they may allocate different expert networks to each function.

As in the case of the modular architecture with a single gating network, competition among the expert networks of the architecture with multiple gating networks determines how training patterns are allocated during learning. The learning algorithm for the modular architecture with multiple gating networks is identical to that described in Section 2 for an architecture with one gating network, with the proviso that the weight modifications are determined independently for each gating network.

Specifically, a modular architecture with multiple gating networks can be thought of as multiple separate modular architectures each having a single gating network. The separate architectures share expert networks although their gating networks gate different sets of the expert networks’ output components. Consequently, the output components of the expert networks that are not gated by a gating network do not participate in the weight-modification process for that gating network. This implies that for each gating network there is a separate process for determining when the performance of the architecture has significantly improved, and this process is identical to that used in architectures with single-gating networks (Equation 5), except that it only depends on the error over the output components gated by the gating network. This results in different values $\lambda_{WTA}$ and $\lambda_{NT}$ for each gating network, and, therefore, different error functions for each gating network. The error functions are given by Equation 7 with the substitution of the appropriate values for $\lambda_{WTA}$ and $\lambda_{NT}$.

We trained a specific instance of the modular architecture shown in Figure 7 to perform simultaneously the what and where tasks. This means

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4 Assuming that the output units of the gating network produce binary outputs, the training of each expert network is identical to the case where an external teacher provides desired values for some output units of the expert network and places “don't care” conditions on other output units. At any given time step, let $X$ denote the set of an expert network’s output units whose outputs are multiplied by a gating network output of 1, and let $Y$ denote the set of output units whose outputs are multiplied by a gating network output of 0. During training, a non-zero error vector is backpropagated to the units in $X$, and the zero vector is backpropagated to the units in $Y$. Therefore, the output units in $X$, but not in $Y$, modify their weights and send error information to the hidden units of the network. In this sense, the training of this expert network is identical to the case where desired values are provided for the units in $X$ and “don’t care” conditions are placed on the units in $Y$ (see Jordan, 1986, for a discussion of “don’t care” conditions and spatial crosstalk).
that at each time step the architecture's output pattern should correctly identify both the object and its location in the current input pattern. The specifications of this modular architecture are summarized in Table 3. It has two expert networks and two gating networks. Both expert networks have 25 input units and 18 output units. The input units encode the 5 x 5 retinal matrix. Because both tasks are to be performed simultaneously, the architecture does not receive a task specifier. The first 9 output units of each expert network encode the nine possible objects, and the second 9 output units encode the nine possible locations. The first expert network is a two-layer network with 36 hidden units, and the second expert network has a single layer of modifiable weights, that is, it has no hidden units. The two gating networks are extremely simplified: Each has 2 output units but no hidden units and no input units. Each gating network, therefore, just consists of 2 output units, each with a single bias weight. The first gating network gates the first 9 output components of the expert networks, and the second gating network gates the second 9 output components of the expert networks.

There are two ways that this modular architecture might successfully learn to perform the what and where tasks. One possibility is that the multilayer expert network learns to perform both tasks, and the second possibility is that the multilayer expert network learns to perform the what task whereas the single-layer expert network learns to perform the where task. For the reasons previously described, the second possibility is the better solution.

Our first simulations of the architecture on the two tasks produced disappointing results. The architecture consistently allocated the single-layer expert network to both tasks. Because the what task is not linearly separable, the architecture did not correctly perform this task. Further analysis of the modular architecture's learning rule reveals the reasons for this behavior. Recall from Section 2 that the architecture's current performance is compared with its past performance at each time step. If the performance has significantly improved, then the weights of a gating network are modified so that the gating network output corresponding to the winning expert network increases toward 1, and the outputs corresponding to the losing expert networks decrease toward 0. If the current performance does not show significant improvement, then a gating network's weights are modified so that all of its outputs are moved towards a neutral value.

In the initial training runs, it turned out that the single-layer expert network approximated the what function more quickly than did the multilayer expert network. Therefore, the output of the first gating network corresponding to the single-layer expert network approached 1, and the output corresponding to the multilayer expert network approached 0. However, because the what task is not linearly separable, the performance of the single-layer expert network on the what task could not improve past a certain low level. Consequently, we expected the outputs of the gating network eventually to approach the neutral value and then change so as to reallocate tasks appropriately to expert networks, but this did not happen. The architecture failed to reallocate tasks because even the low level of performance achieved by performing both tasks with the single-layer expert network was better than the performance attainable by nearby weight values. The gating network consistently became trapped in this local error minimum.

Rather than devising a mechanism to allow the architecture to escape from this kind of local minimum, we modified the learning process in order to make the architecture less likely to become trapped in this way. During training we varied the contribution to the gating network error function (Equation 7) of the term responsible for making the outputs of the gating network approach the neutral value 1/n, where n is the number of expert networks. This term, the fourth term in Equation 7, contains the factor $\lambda_{NT}$ which is nonzero only when the architecture's performance has not significantly improved. Instead of setting $\lambda_{NT}$ for a gating network to 1 when performance has significantly improved, we initialized it to a value greater than 1 at the start of training and slowly decreased it to 1 during training. Larger values of $\lambda_{NT}$ increase the tendency of the gating network outputs to remain near the neutral value and therefore prolong the period of training before the expert networks specialize. This gives expert networks unable to compete in terms of initial rates of learning—but which may be better in terms of eventual performance—the chance to exert their superiority. As a result, an expert network that learns a task quickly at first, such as the single-layer network in the what task, does not necessarily become allocated to that task.

When modified in this manner, the modular architecture with two gating networks that we simulated, consistently learned to allocate the multilayer expert network to the what task and the single-layer expert network to the where task. As the outputs of the expert networks came to be gated in an appropriate manner by the gating networks, the degree of conflict in the

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1. At best, the single-layer expert network can correctly perform 54% of the what task's input–output pairs.

2. Recall that when there are multiple gating networks (the case we are discussing) there is a different $\lambda_{NT}$ for each gating network.

3. The effect of altering $\lambda_{NT}$ is to alter the step size of changes in gating network weights that occur when the architecture's performance has not significantly improved.
training information received by the output units of the expert networks decreased, that is, spatial crosstalk decreased. However, we cannot report that, as a result of decreased spatial crosstalk, the modular architecture learned the what and where tasks more rapidly than did the single-network system studied by Rueckl et al. (1989). It is not surprising that prolonging the period before the expert networks specialize considerably slows learning, but this may be necessary for the modular architecture to allocate networks selectively to tasks based on the suitability of the networks’ topologies.

In summary, we have studied the performances of single networks and modular architectures on the what and where tasks of Rueckl et al. (1989). One major goal has been to demonstrate the modular architecture’s ability to perform function decomposition in the sense of learning to allocate different networks to learn different functions. A second goal has been to suggest that the architecture tends to allocate to each function a network with a topology that is appropriate to that function. We have also shown that, at least in the case presented here, the modular architecture is robust in the presence of temporal crosstalk.

5. TASK DECOMPOSITION AND NETWORK ARCHITECTURES

The previous section reported the ability of the modular architecture to perform task decomposition on the what and where vision tasks. In this section, we consider some domain-independent issues concerning task decomposition and discuss similarities between task decomposition and generalization as implemented in connectionist systems.

Because function approximation is an underconstrained problem, networks with too many degrees of freedom may not generalize as desired (e.g., Denker et al., 1987; Le Cun, 1989; Poggio & Girosi, 1989). One approach to this problem is to use domain knowledge to design a network architecture that is appropriately restricted in the types of functions that it can implement. Such an architecture should generalize in the desired manner. Experience with the modular architecture described here has shown that a similar situation exists with regard to task decomposition. Because function decomposition is an underconstrained problem, there are many possible decompositions of a task into simpler tasks. Thus, modular architectures with too many degrees of freedom may not decompose a task as desired. If there are reasons to prefer one decomposition over another, then it is necessary to use domain knowledge in order to design a modular architecture that is appropriately restricted in the types of functions it can compute. Such an architecture should decompose a task in the desired manner.

The design of an appropriate architecture, whether for generalization or task decomposition, may use prior knowledge about the task the system will be required to perform. A strength of modular architectures is that their structures are well suited for incorporating prior knowledge to bias the nature of the decompositions to be formed. For example, in many tasks there is a natural distinction between information to be processed and information that sets the context for processing. The experimenter may know that when one or more of the (context) inputs remain constant, the mapping from the remaining inputs to the desired outputs is relatively easy to compute. The distinction between the context inputs and the remaining inputs can form the basis for deciding how to divide input information between gating networks and expert networks.

Another way prior knowledge can be incorporated into the design of a modular architecture is that known properties of the function to be approximated can provide constraints on the design of the expert networks. The design of the expert networks, in turn, biases the nature of the decomposition discovered by the modular architecture. This is particularly true when the repertoire of expert networks consists of networks with different characteristics. For example, the function to be approximated may be known to contain a linear portion and a nonlinear portion. In this case, suitable expert networks are easy to design. In general, different expert networks may be designed to possess different topologies, initial weights, activation functions, step sizes, error functions, and so on. In addition, different expert networks may receive different input variables or perhaps different representations of the same input variables. Note that we do not advocate providing the modular architecture with a large number of different expert networks. Rather, the experimenter should judiciously design a small set of potentially useful expert networks where the potential utility of an expert network is evaluated using domain knowledge. Indeed, if there is sufficient knowledge of the task, some or all of the expert and gating networks can be individually trained independently of the rest of the architecture (Jacobs, 1990; Hampshire & Waibel, 1989).

6. CONCLUSIONS

The notion of modularity has been found to be of considerable utility in cognitive science, particularly in the study of language and vision. Not only have modular theories been found to be more parsimonious and easier to understand than nonmodular theories, but also the predictions of modular theories have in many cases been verified (Freedman & Forster, 1985). Modularity is also indispensable in the design and analysis of complex systems in engineering. We believe that the virtues of modular systems cited in the literature of these areas are also relevant to the problem of learning in connectionist networks. In particular, we have argued that if a task can be decomposed into subtasks, each of which has its own idiosyncratic properties, then the learner should itself be a decomposable system in which dis-
distinct system resources ("experts") are allocated to distinct subtasks. Such a learning system will, in general, be more robust, more efficient, and will generalize better than a nonmodular system.

Although domain knowledge may be useful in suggesting an a priori decompositon of a task, the boundaries between subtasks are rarely explicitly marked in the data presented to the learner. Moreover, the optimal allocation of experts to subtasks depends not only on the nature of the task, but also on the nature of the learner. For these reasons we have argued that the problem of allocating experts to subtasks is itself part of the learning problem. Even if domain knowledge is used in designing the initial structure of the modular architecture, it is still necessary for the system to discover which experts to assign to which training instances.

This article has examined the development of modular connectionist systems on the basis of a simple underlying principle. Although many types of interaction between components of a system are possible, the modular architecture presented here makes use of competition to induce a task decomposition. The competition allows experts to specialize as well as to extend their domain. We believe that the competition between experts is the essential feature of the approach presented here and should serve as a useful point of departure for the further development of algorithms for learning in modular systems.

REFERENCES


APPENDIX

This appendix provides details about simulations not included in the main body of this article.

Input Values
The task specifier was set to $-1$ for the what task and to $1$ for the where task.

Training
The weights of all systems were updated at each time step. Desired output values of $.1$ and $.9$ were used instead of $0$ and $1$.

Activation Functions
The hidden and output units of all systems used activation functions that include the logistic function with asymptotes at $0$ and $1$.

Initial Weights
The weights of the single networks and the expert networks were initialized with values randomly selected from a uniform distribution over the interval $[-\frac{1}{2}, \frac{1}{2}]$. The weights of the gating network were initialized to $0$.

<table>
<thead>
<tr>
<th>APPENDIX TABLE</th>
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<tbody>
<tr>
<td>Parameter Values Used in Temporal Crosstalk Experiments</td>
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<tr>
<td>System</td>
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<tr>
<td>--------</td>
</tr>
<tr>
<td>Random Training</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
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<tr>
<td>3</td>
</tr>
</tbody>
</table>

Parameter Values

**Temporal Crosstalk Experiments.** For the single networks trained with the backpropagation algorithm, we used the step sizes and momentum that roughly give the best performance. These values are listed in the above table.

For some of the parameters of the modular architecture, we searched for the values that gave the best behavior, and for others, we didn’t. Specifically, we did not attempt to optimize the step size and momentum of the gating network and $\alpha$ used in Equation 4. These parameters had the values $0.01$, $0.0$, and $0.2$, respectively. In addition, although each expert network may have its own step size and momentum, the same values were used for all expert networks. Thus, the only parameter values that we attempted to optimize are the step size and momentum used by all expert networks and $\gamma$ used in Equation 5. These values are listed in the above table.
Spatial Crosstalk Experiment. Because no comparisons were performed in this set of experiments, we did not attempt to locate the optimal parameter values. Thus, we provide the values that were used, not the values that are best. For the expert networks, the step size and momentum were 8.0 and 0.0. For the gating networks, the step size was 0.01, momentum was 0.0, \( \alpha \) was 0.2, and \( \gamma \) was 0.75. When the system's current performance was significantly better than its past performance, \( \lambda_{NT} \) was set to 0. Otherwise, \( \lambda_{NT} \) was 10 for Epochs 1–400, 8 for Epochs 401–500, 6 for Epochs 501–600, 4 for Epochs 601–700, 2 for Epochs 701–800, and 1 for Epochs 801–900.