NEURAL REPRESENTATIONS OF UNCERTAINTY
the sampling hypothesis

MÁTÉ LENGYEL

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PERCEPTUAL UNCERTAINTY

contrast / brightness

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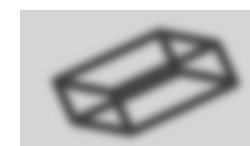
contrast / brightness

3D→2D projection



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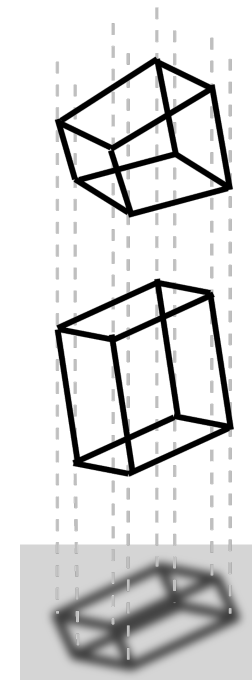

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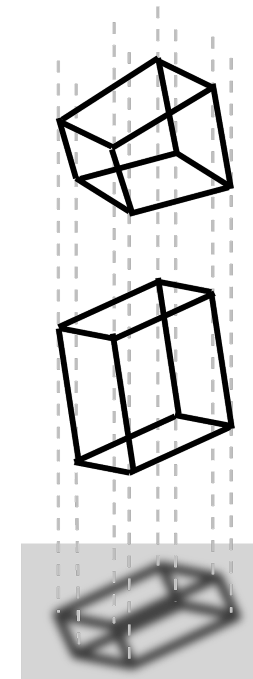
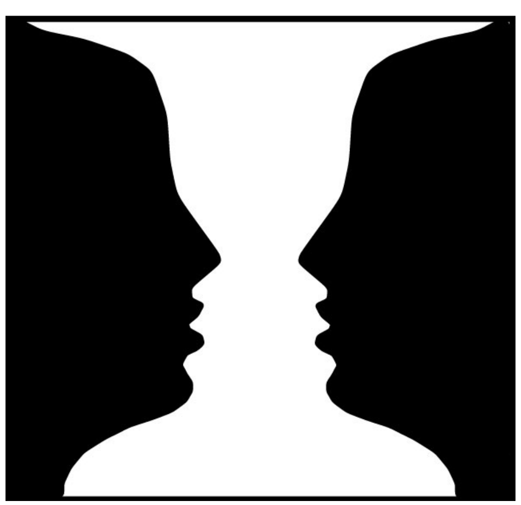


multiple interpretations: bistability



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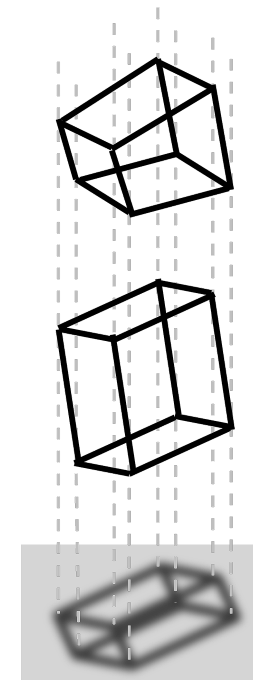

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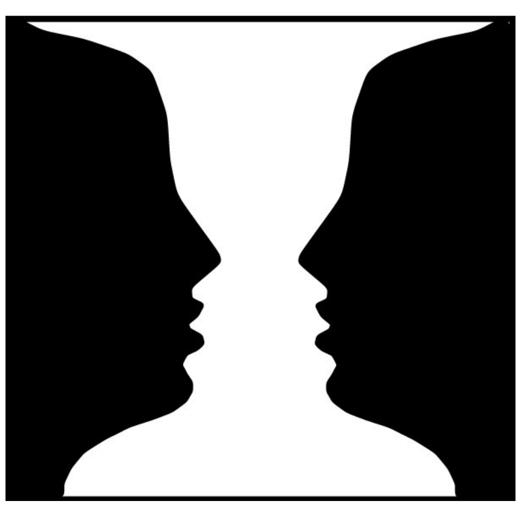
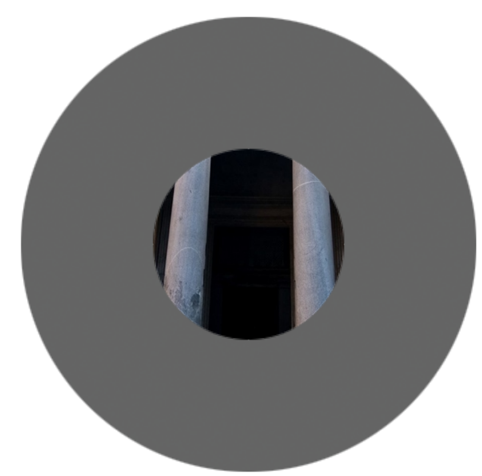


aperture problem: incomplete information



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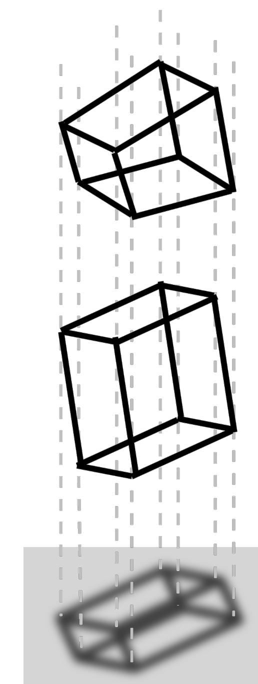

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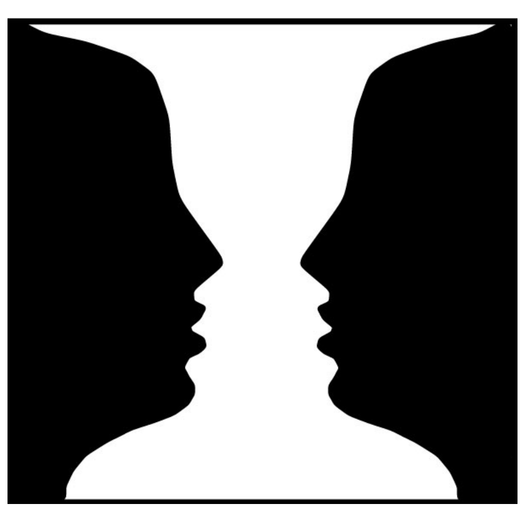

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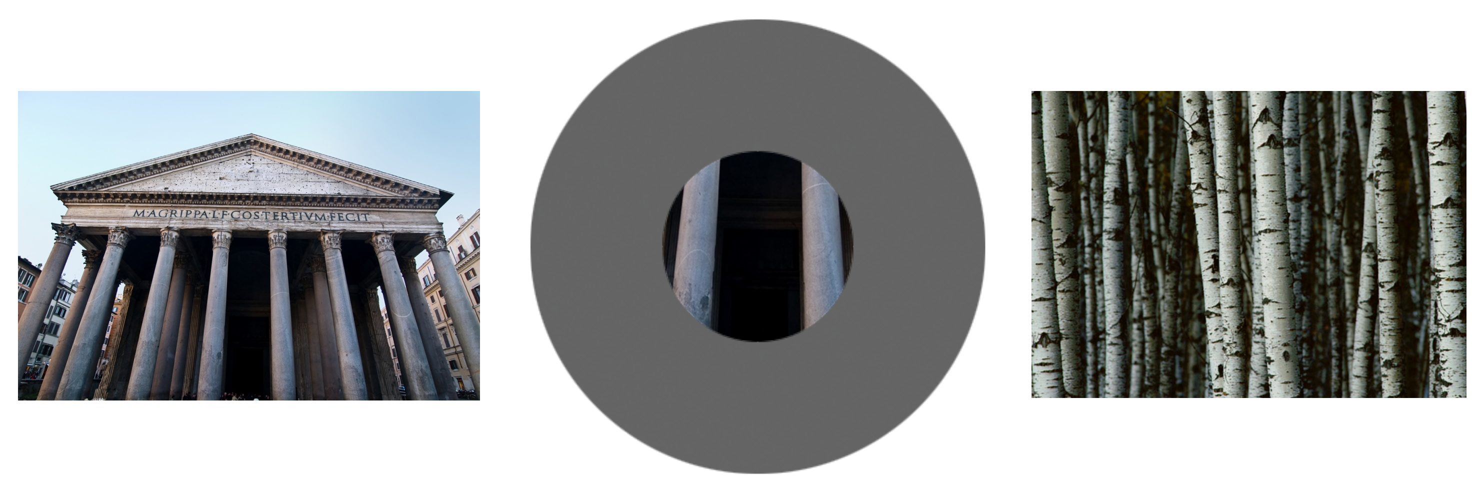

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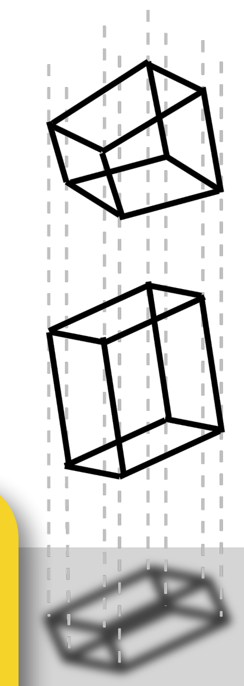
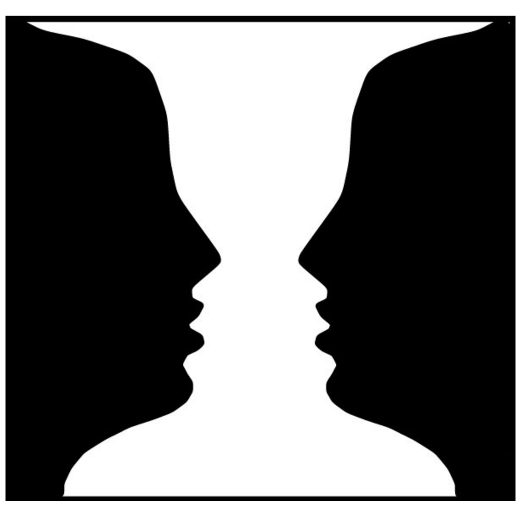


$$P(\text{feature} \mid \text{stimulus})$$

multiple interpretations: bistability



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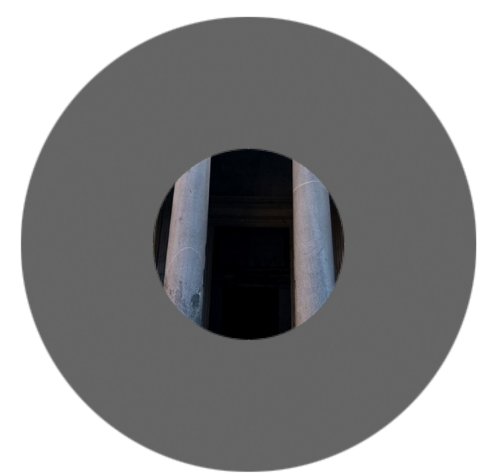
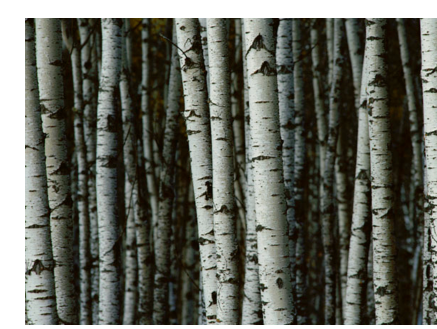
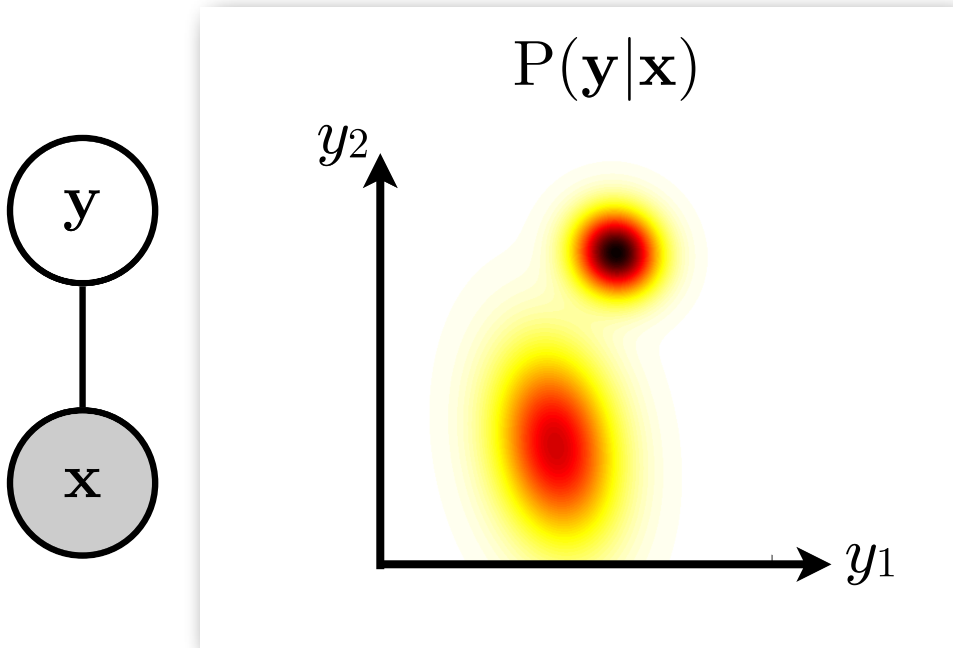




A SIMPLE TAXONOMY OF PROBABILISTIC REPRESENTATIONS

probability distribution



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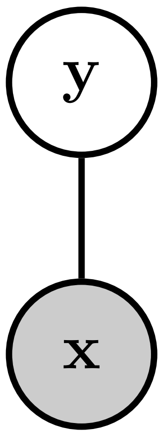
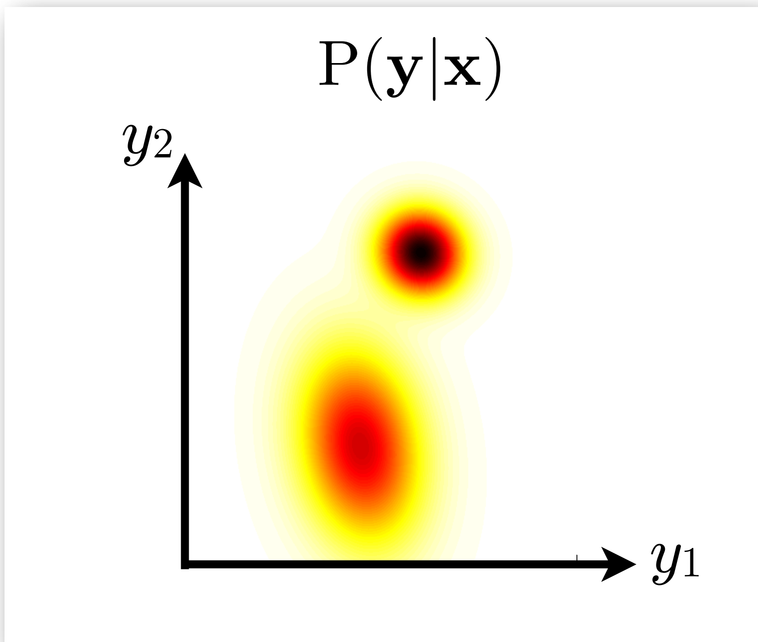
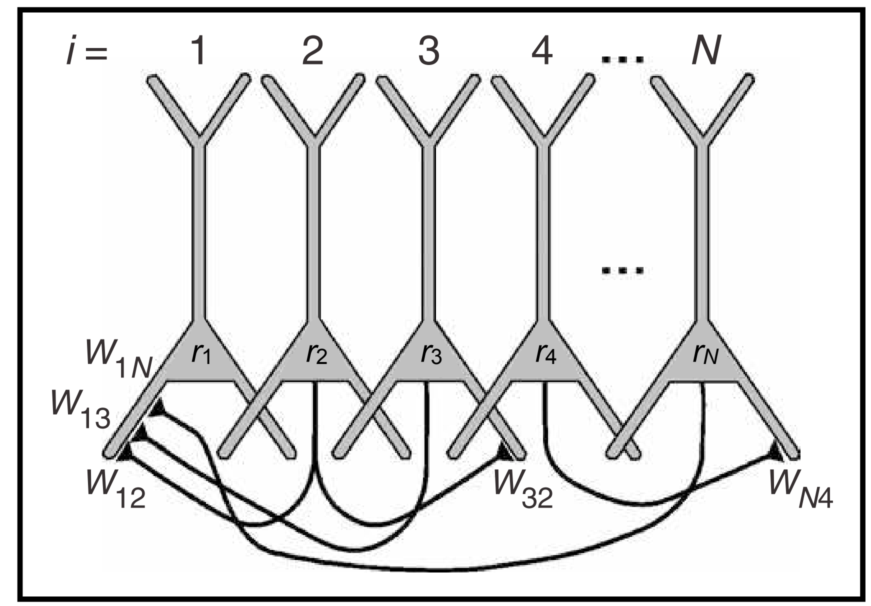
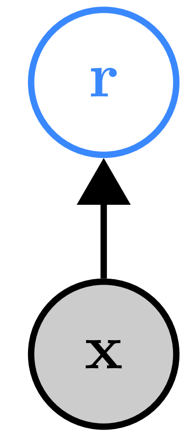




spatio-temporal neural activity patterns

$\mathbf{r}(t,\mathbf{x})$





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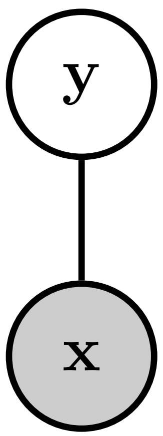

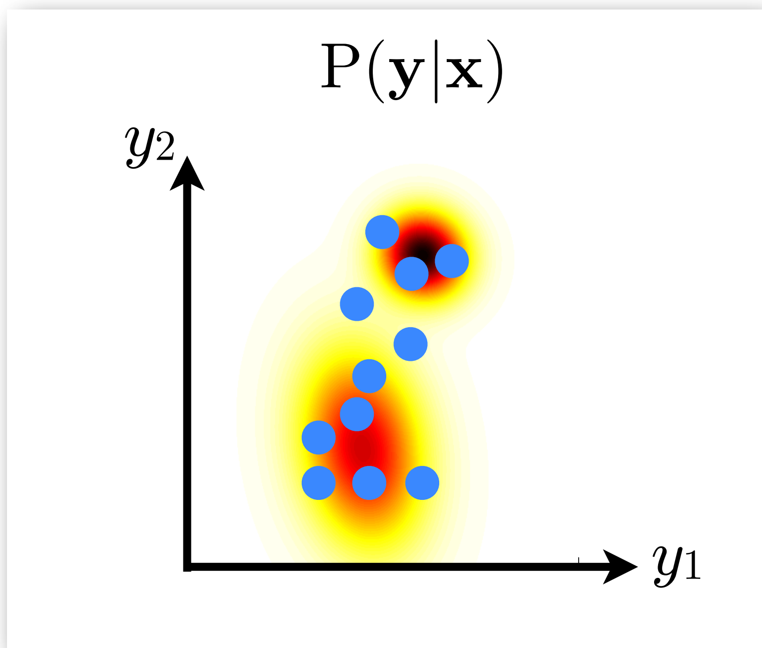

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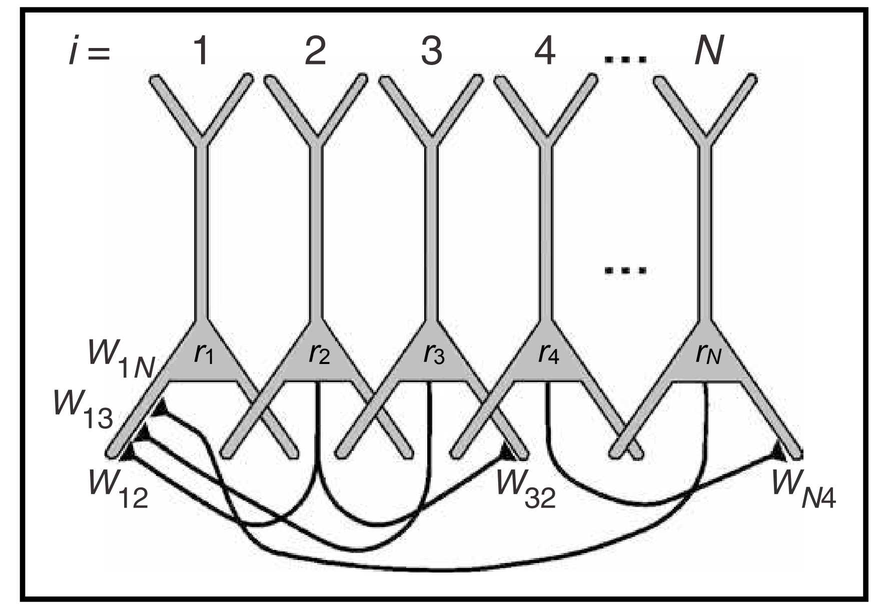

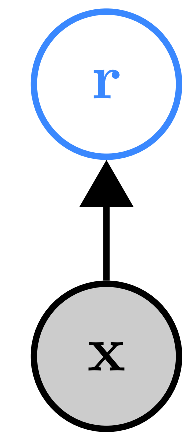

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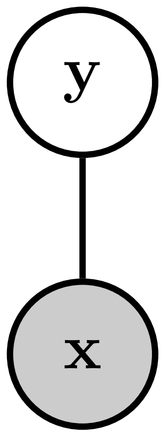

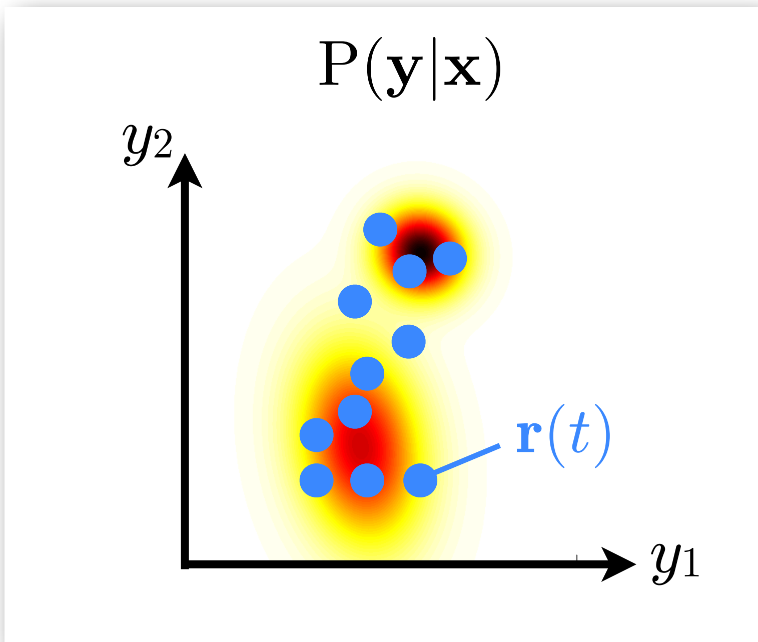

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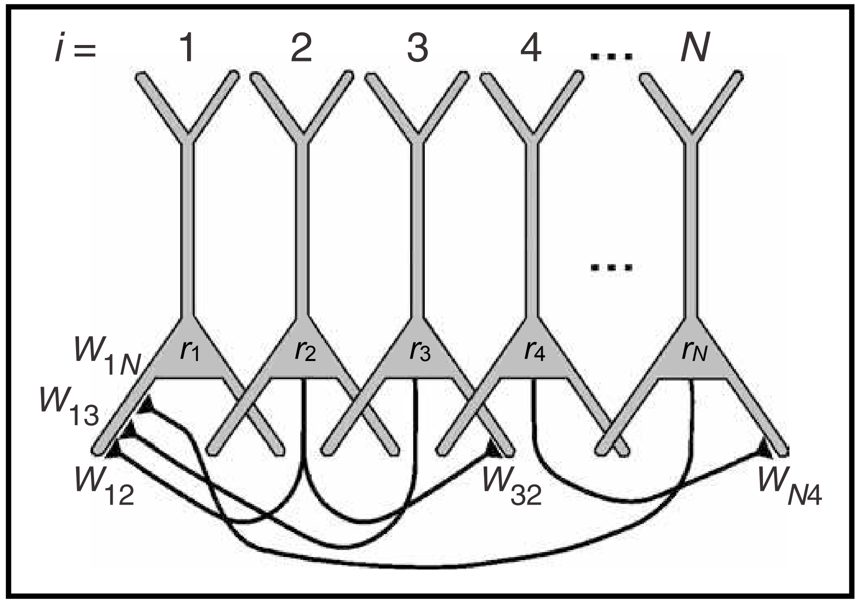

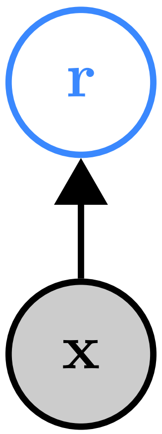

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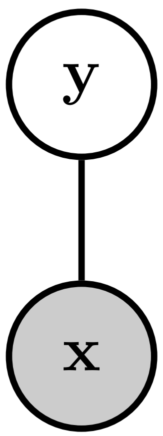
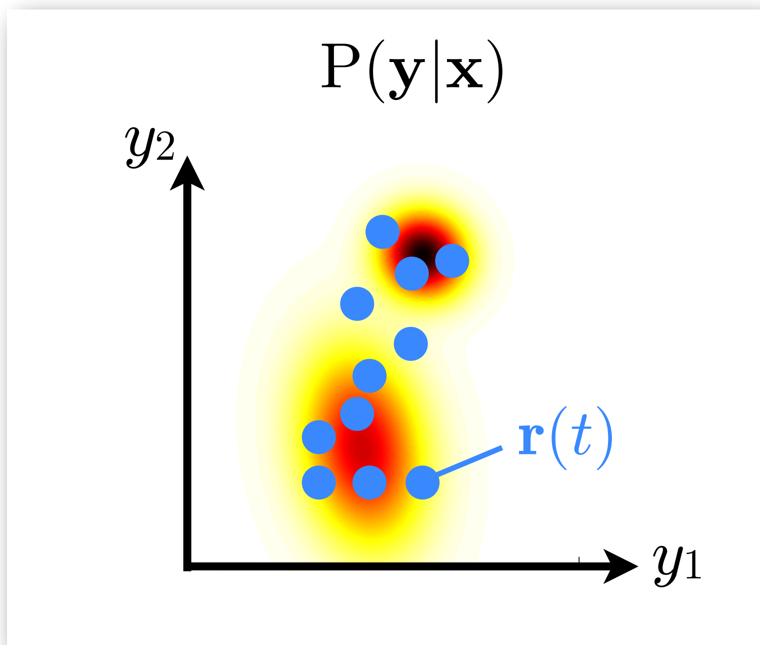
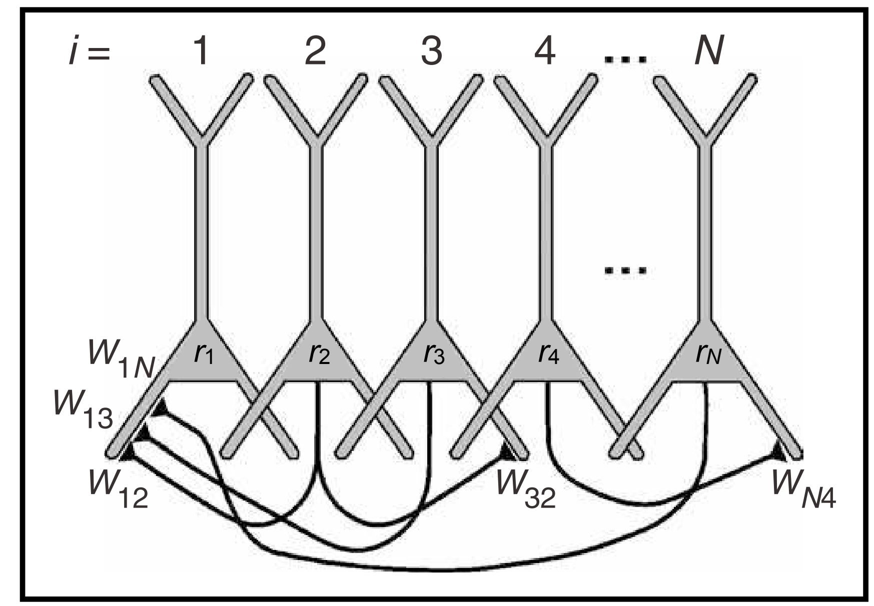


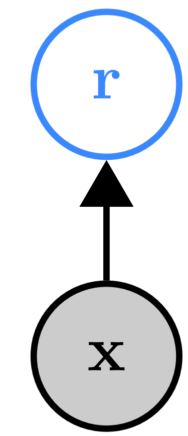


sampling-based

$\mathbf{r} \sim \mathrm{P}(\mathbf{y} = \mathbf{r}|\mathbf{x})$

$\mathbf{r}(t,\mathbf{x})$





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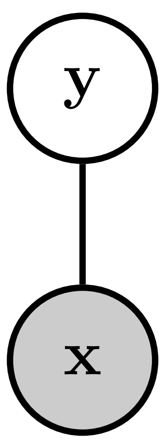

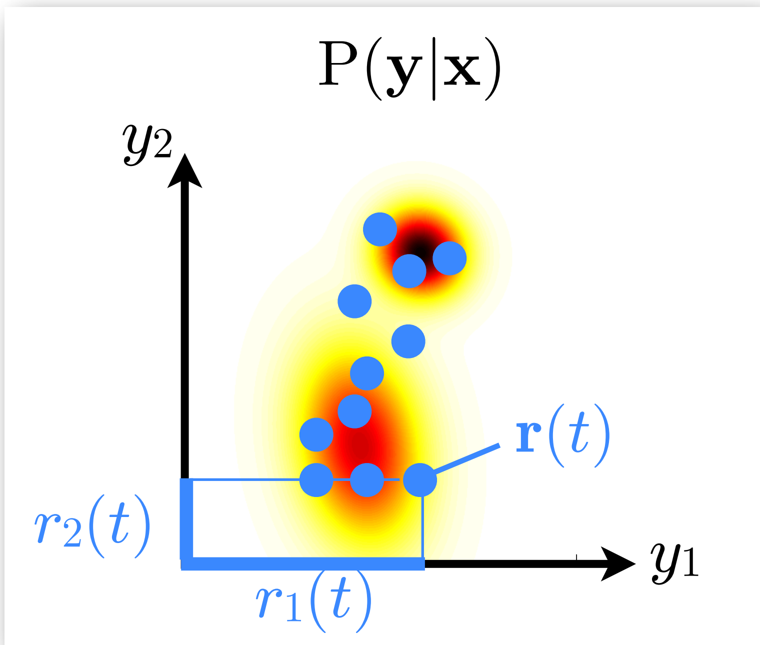

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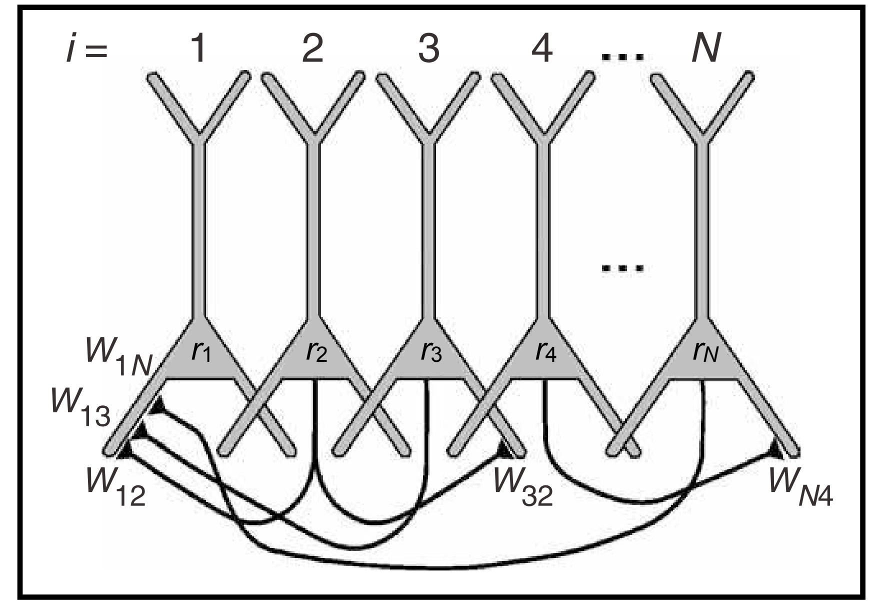

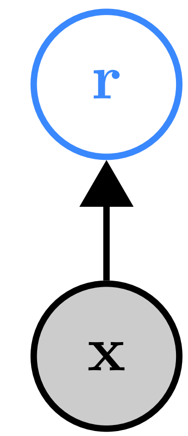

PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY

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VIA SAMPLING

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stimulus

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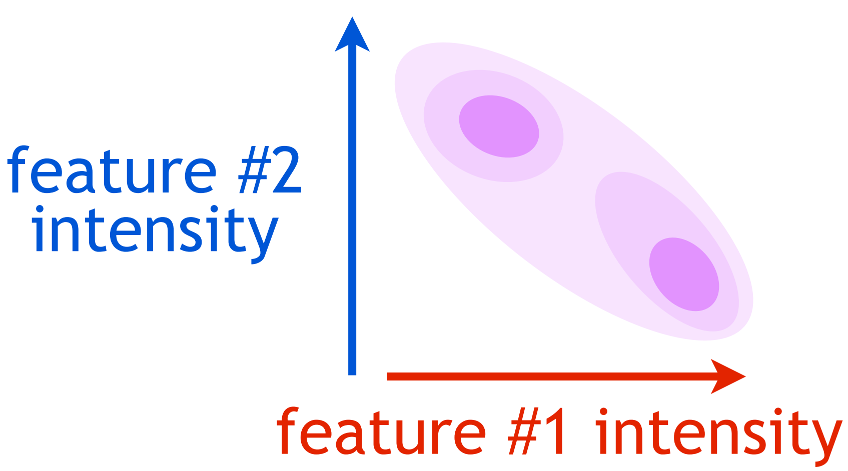
P(feature 1, feature 2 | stimulus)



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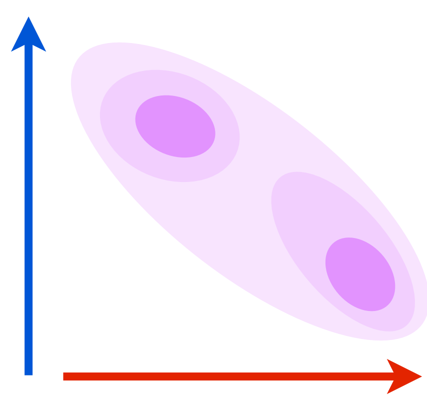

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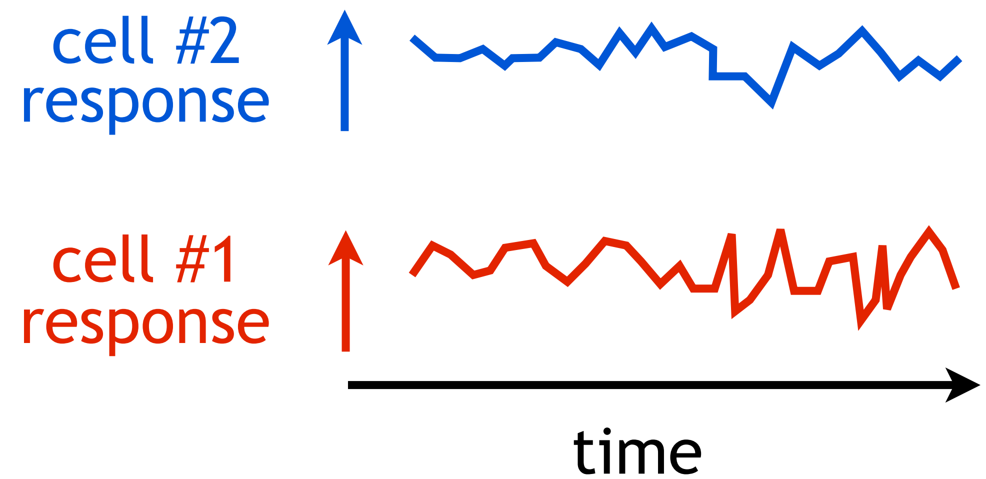

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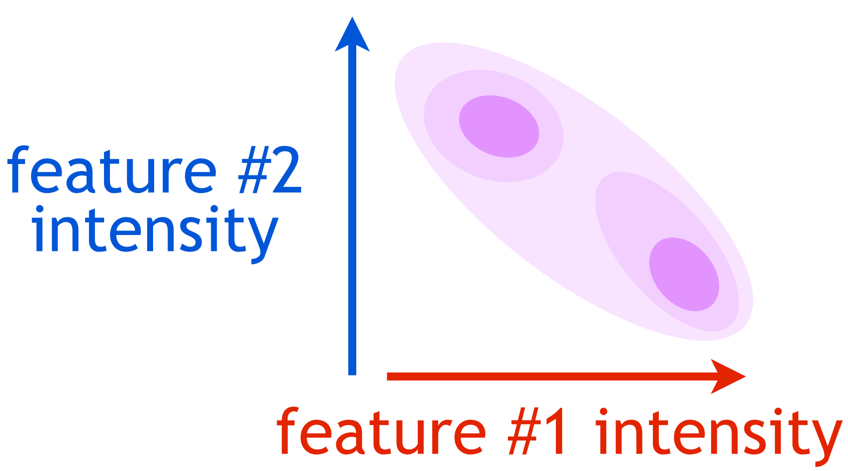

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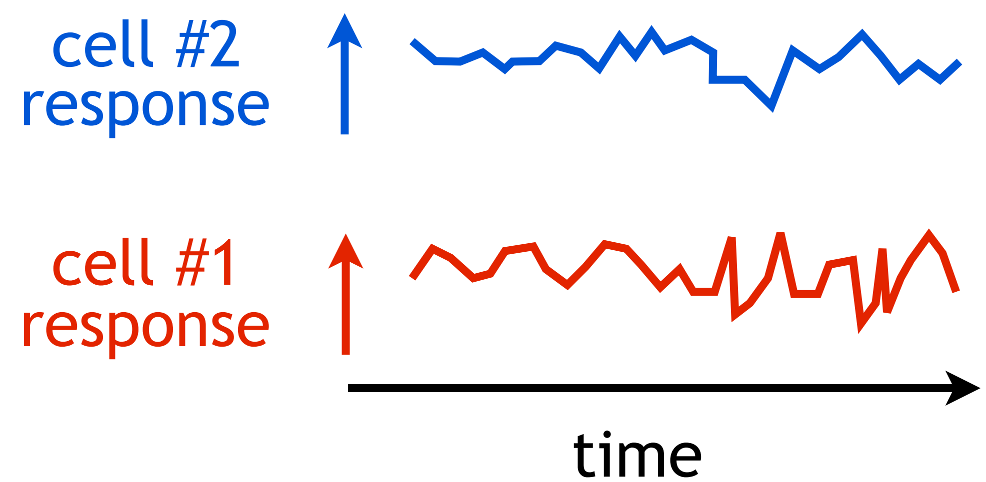

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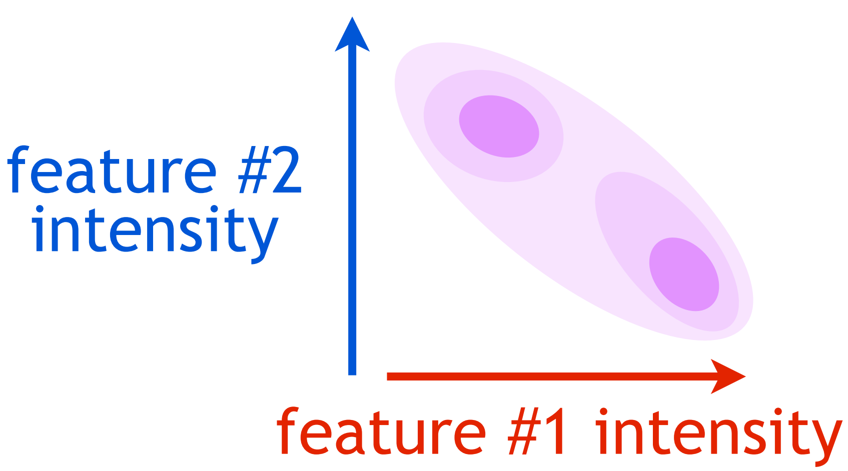

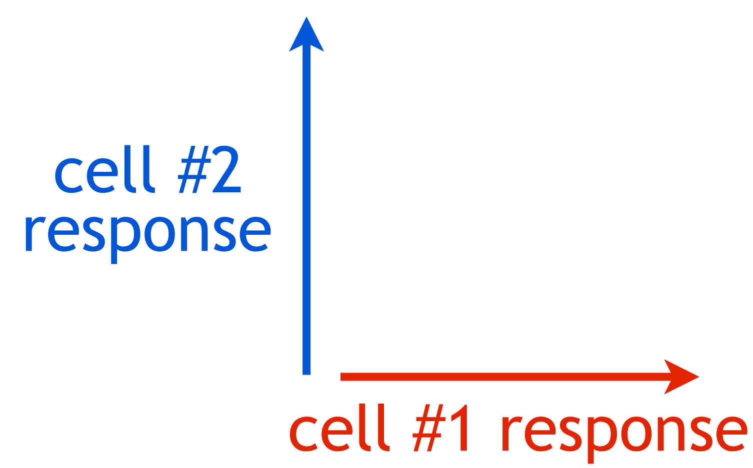

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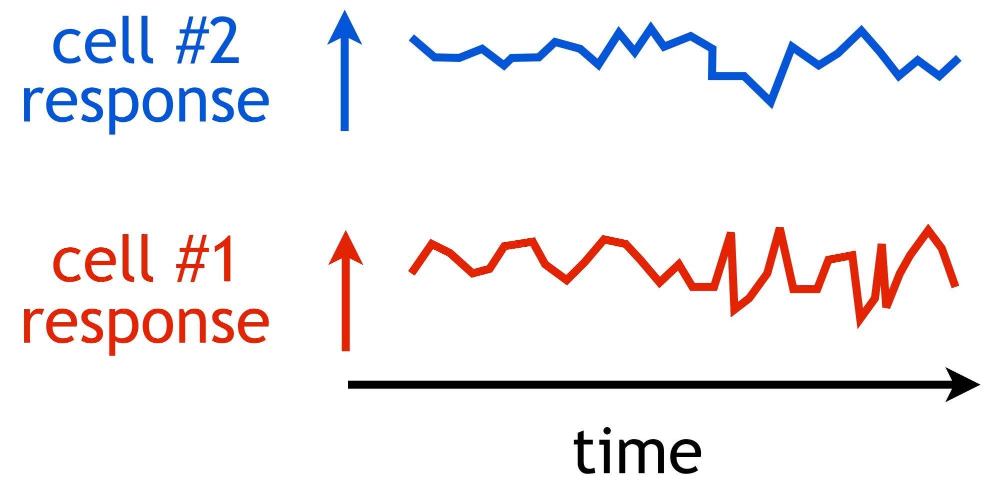

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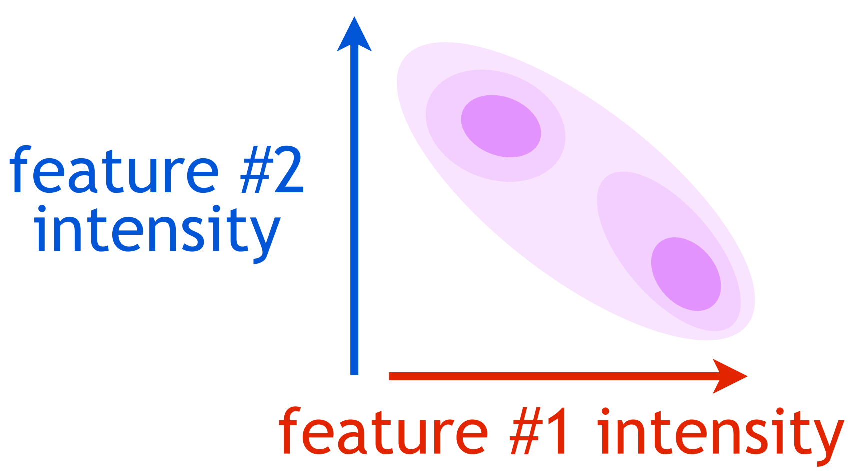

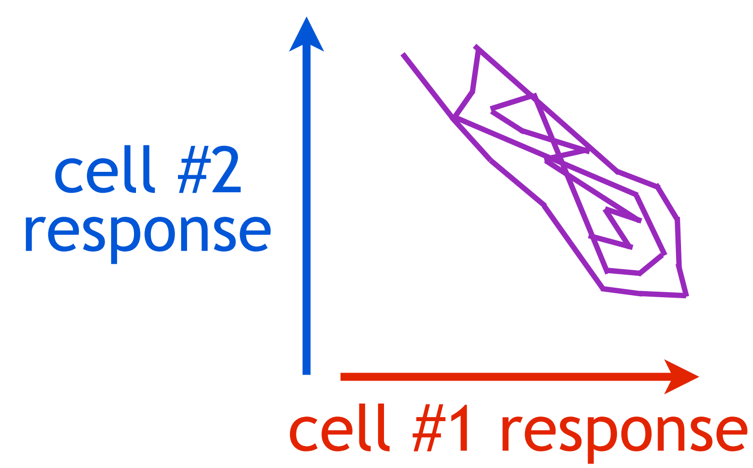

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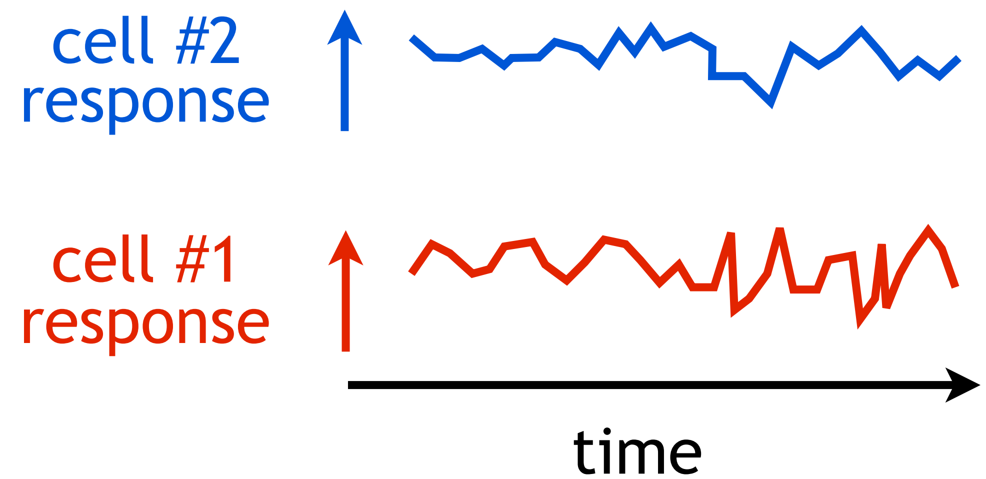
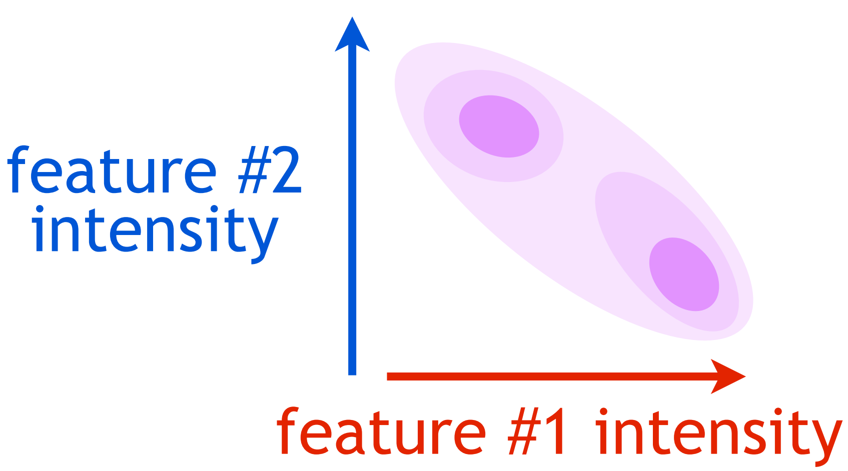
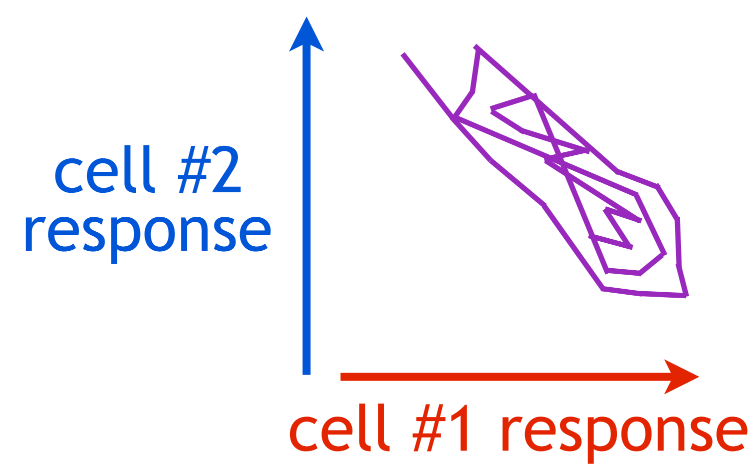
stimulus



P(feature 1, feature 2 | stimulus)

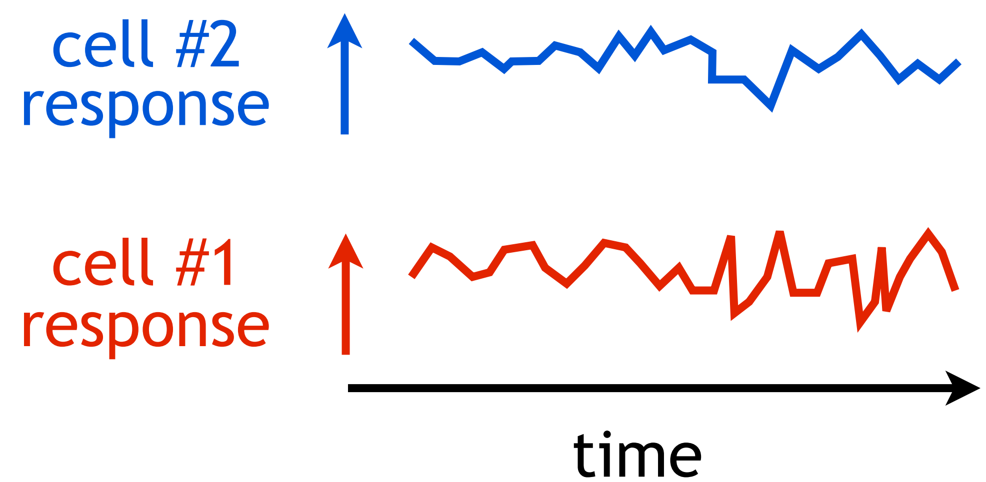


P(response 1, response 2 | stimulus)

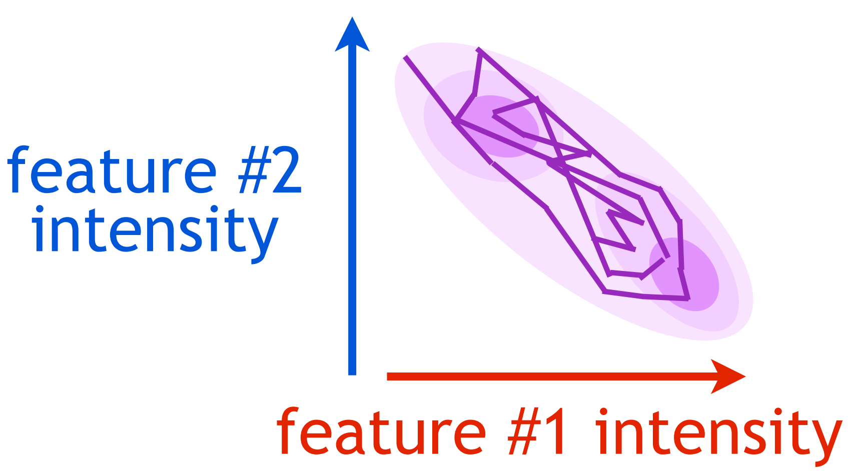


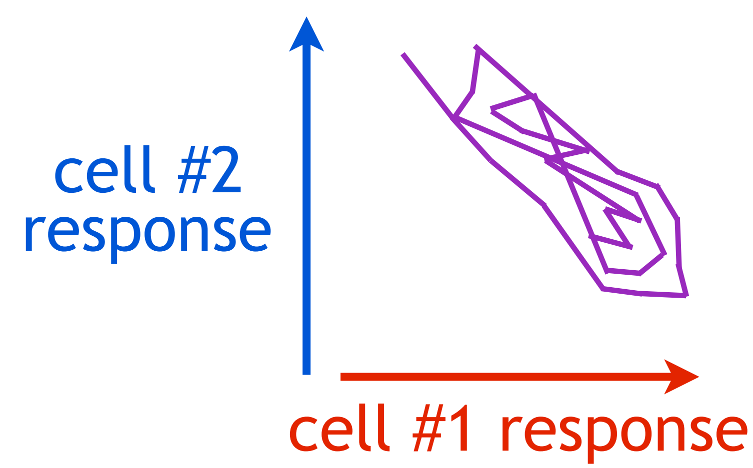
PERCEPTUAL UNCERTAINTY ↔ NEURAL VARIABILITY
VIA SAMPLING

stimulus



P(feature 1, feature 2 | stimulus) = P(response 1, response 2 | stimulus)





Fiser et al, TICS 2010

see also:
Hinton & Sejnowski, PDP 1986; Hinton et al, Science 1995; Dayan 1999; Hoyer & Hyvarinen, NIPS 2003, Lee & Mumford 2003

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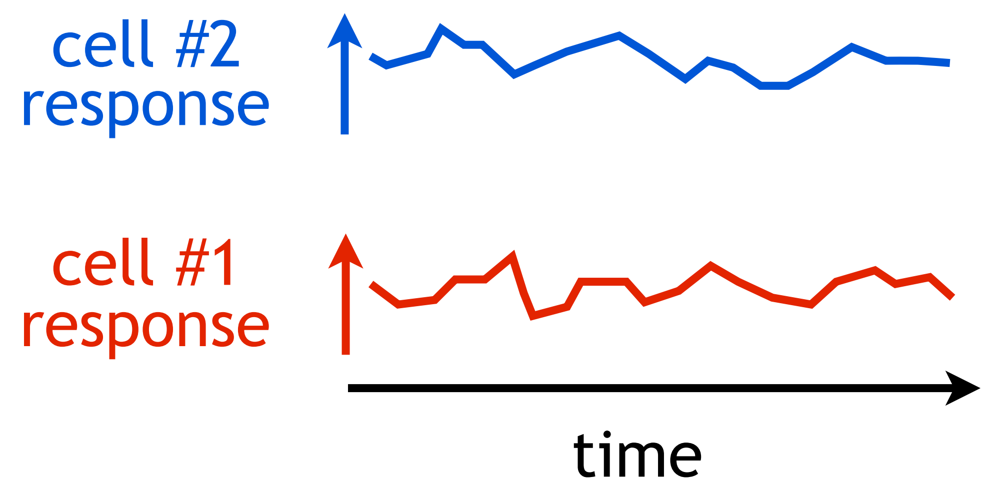
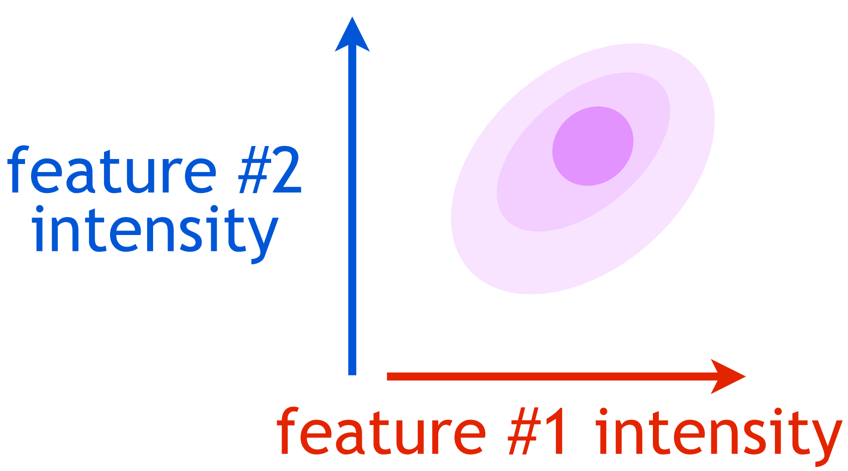
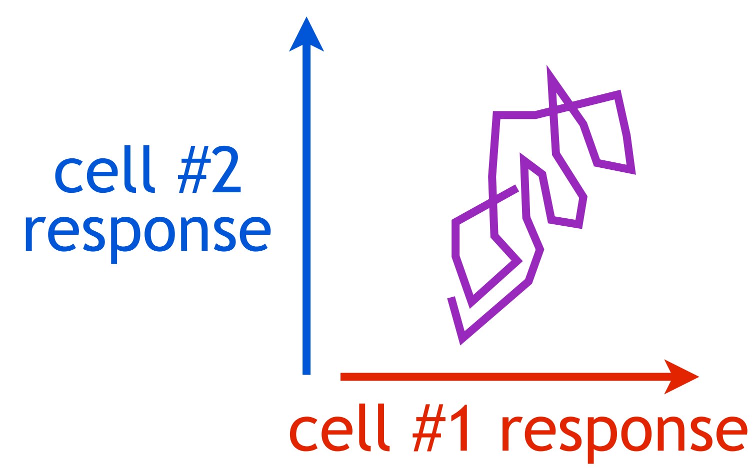


$$P(\text{feature 1}, \text{feature 2} \mid \text{stimulus}) = P(\text{response 1}, \text{response 2} \mid \text{stimulus})$$





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NEURAL RESPONSE DISTRIBUTIONS

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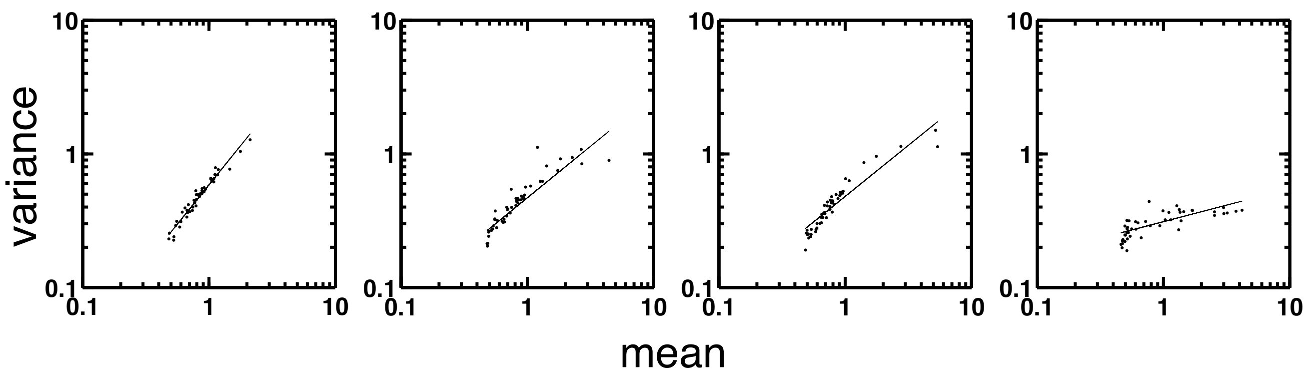
Hoyer & Hyvärinen, NIPS 2003

Poisson-like variability (sparse coding)



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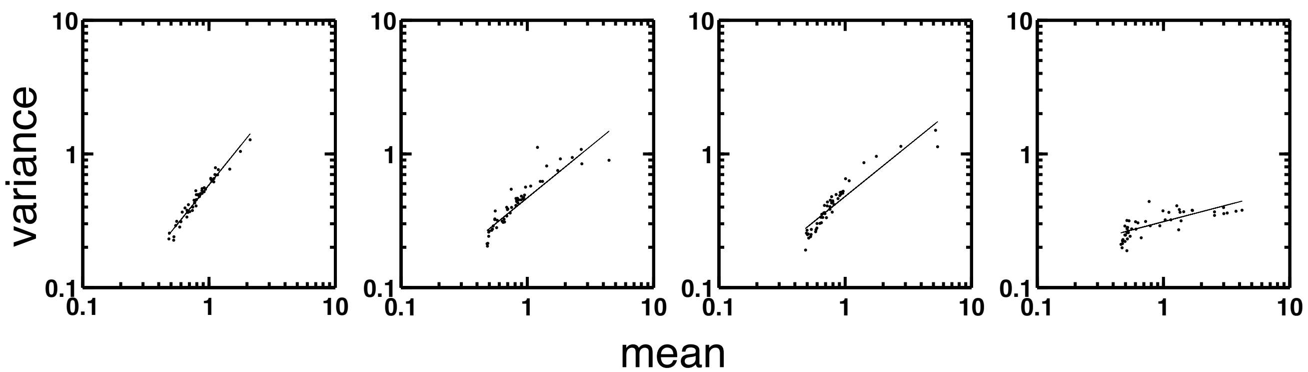
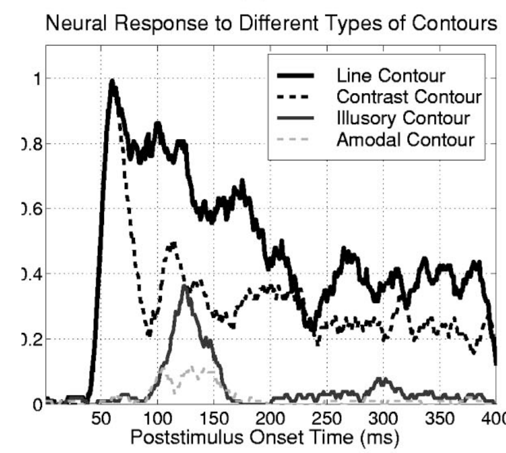


Lee & Mumford, J Opt Soc Am A 2003

feed-back effects
percept-coding neurons



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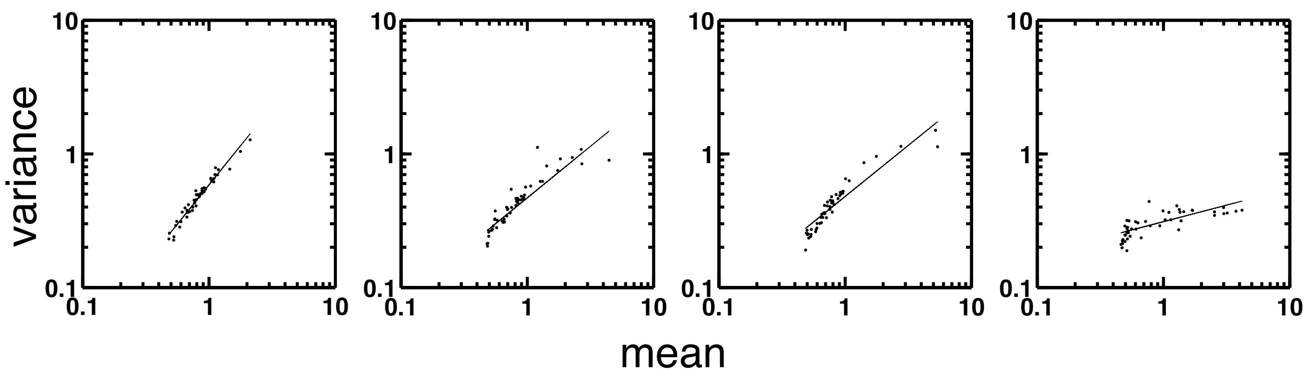

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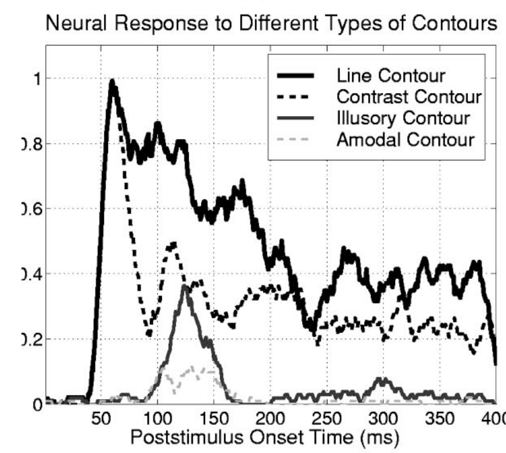
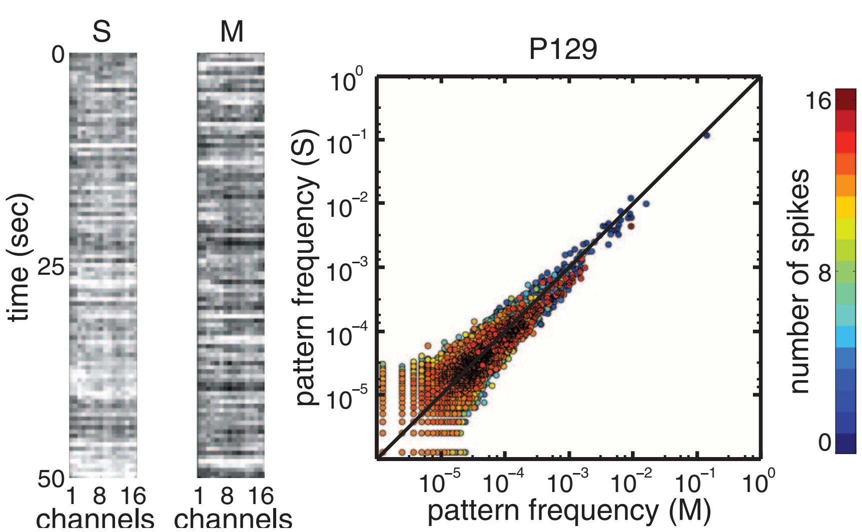


Berkes et al, Science 2011

average evoked = spontaneous
response distributions



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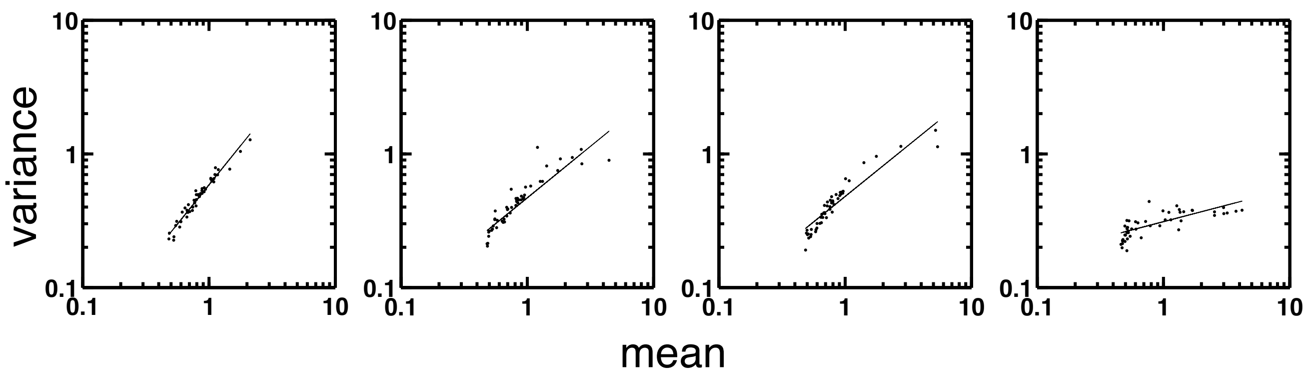

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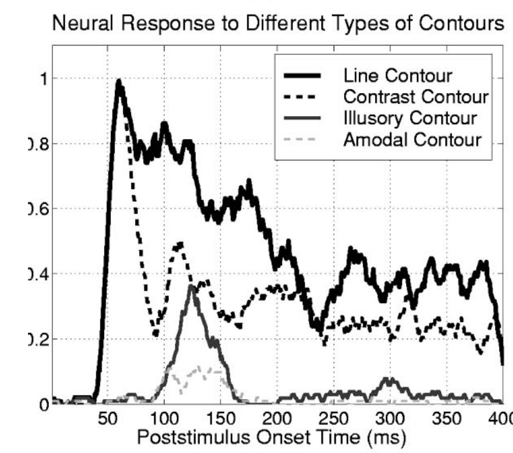

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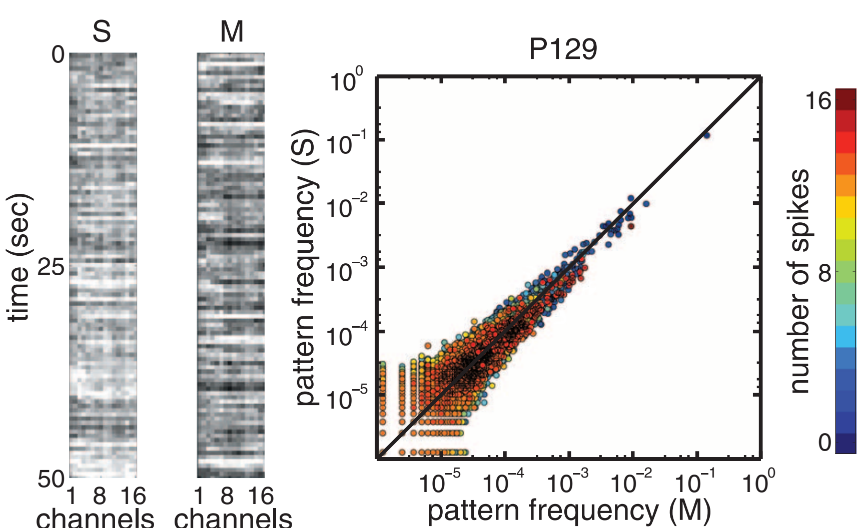
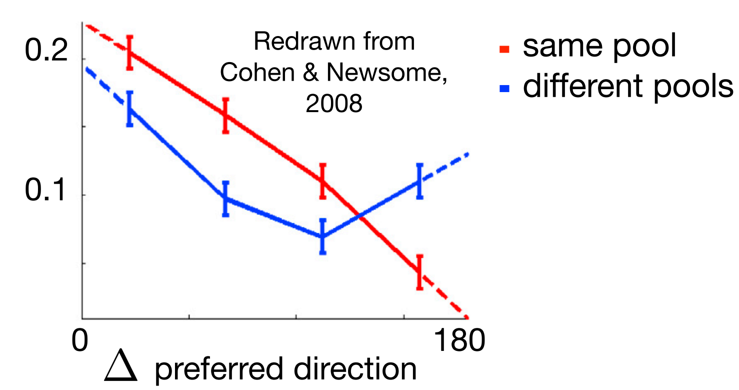


Haefner et al, Neuron 2016

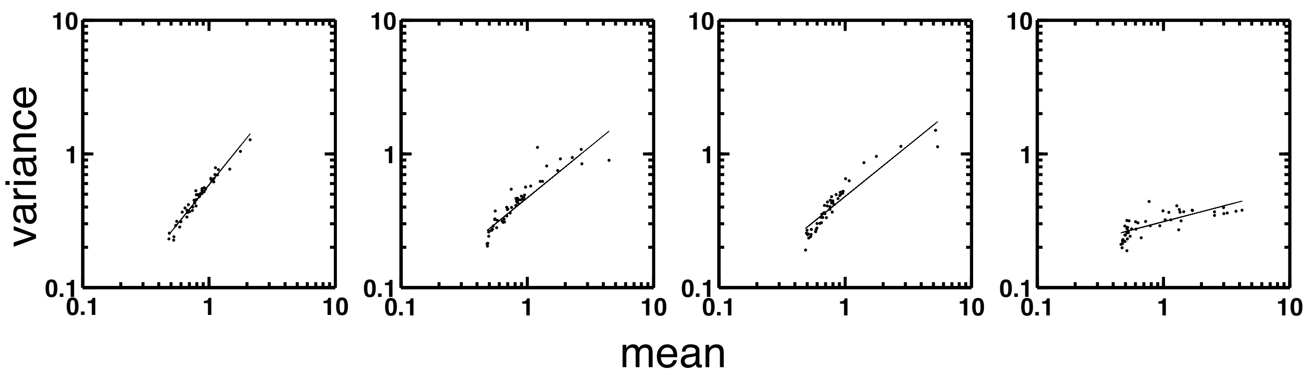
task-dependent changes
in (co)variability



NEURAL RESPONSE DISTRIBUTIONS

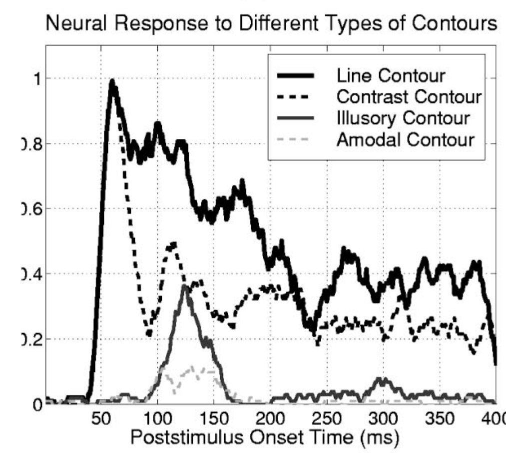
Hoyer & Hyvärinen, NIPS 2003

Poisson-like variability (sparse coding)



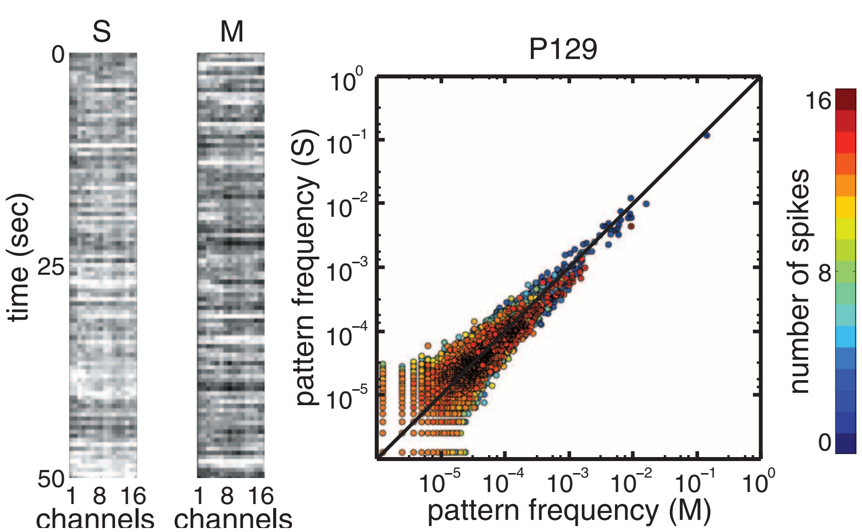
Lee & Mumford, J Opt Soc Am A 2003

feed-back effects
percept-coding neurons



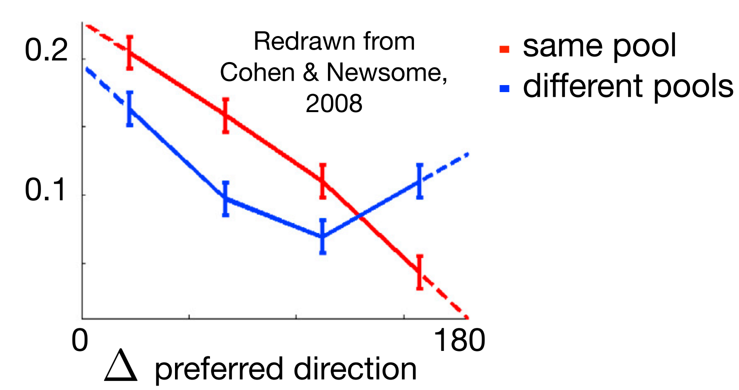
Berkes et al, Science 2011

average evoked = spontaneous
response distributions



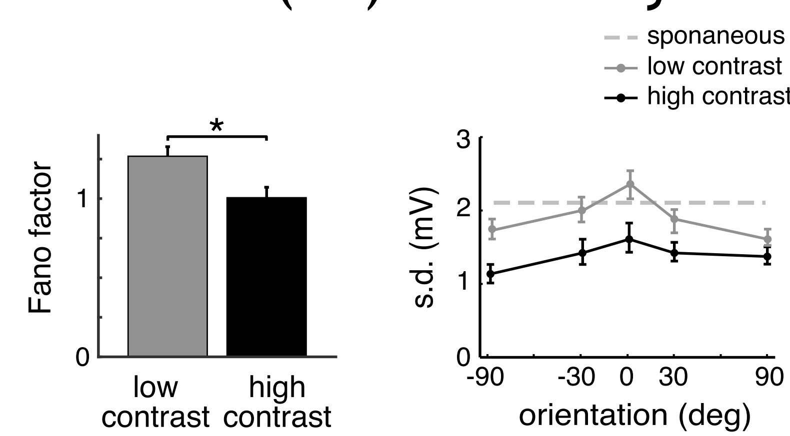
Haefner et al, Neuron 2016

task-dependent changes
in (co)variability



Orbán et al, Neuron 2016

stimulus-dependent changes
in (co)variability



NEURAL RESPONSE DISTRIBUTIONS

see talks tomorrow by

Gergő Orbán

Ruben Coen-Cagli

Ralf Haefner

NEURAL RESPONSE DISTRIBUTIONS

see talks tomorrow by

Gergő Orbán

Ruben Coen-Cagli

Ralf Haefner

SYNAPTIC RESPONSE DISTRIBUTIONS
(“synaptic sampling”)

NEURAL RESPONSE DISTRIBUTIONS

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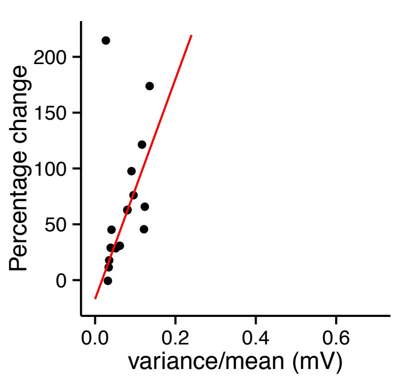
SYNAPTIC RESPONSE DISTRIBUTIONS
(“synaptic sampling”)

Aitchison & Latham, arXiv 2015

plasticity ∝ variability



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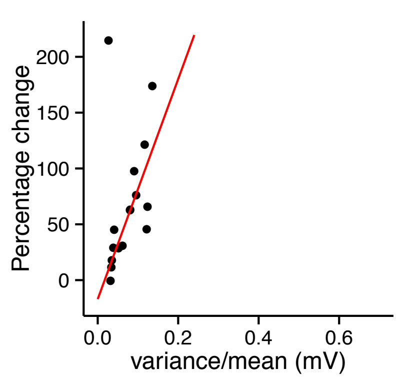


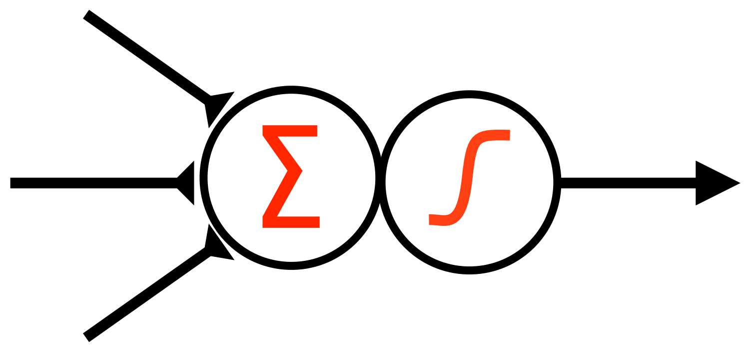
see talk later today by

David Kappel

NEURAL DYNAMICS

NEURAL DYNAMICS

Hinton & Sejnowski, PDP 1986
Hinton et al, Science 1995
Gibbs sampling by binary neurons



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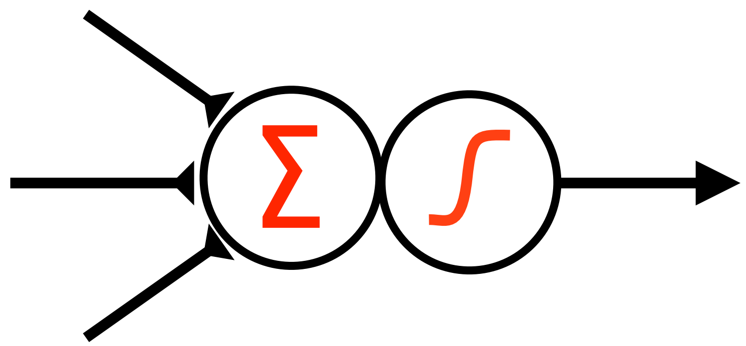
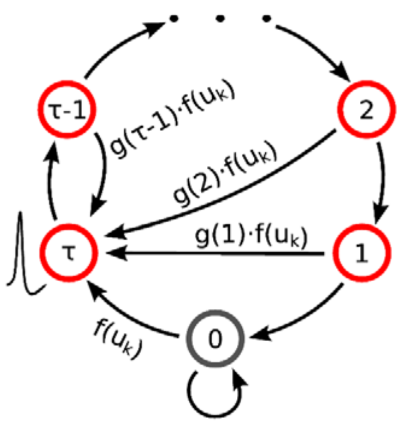


Buesing et al, PLoS Comput Biol 2011

~Gibbs sampling by spiking neurons



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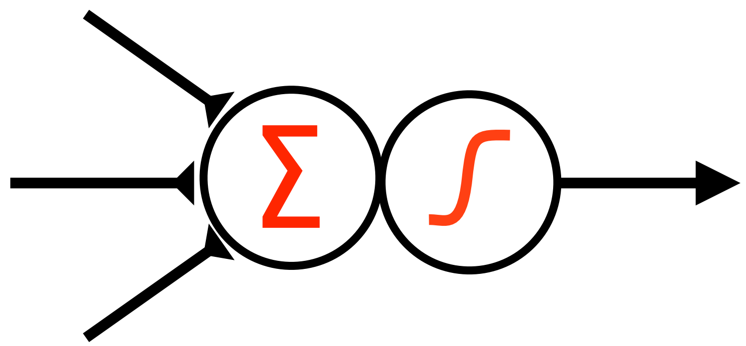

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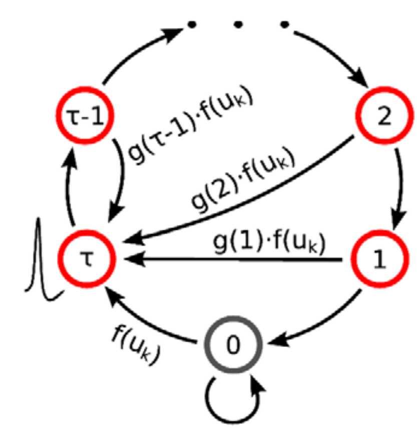
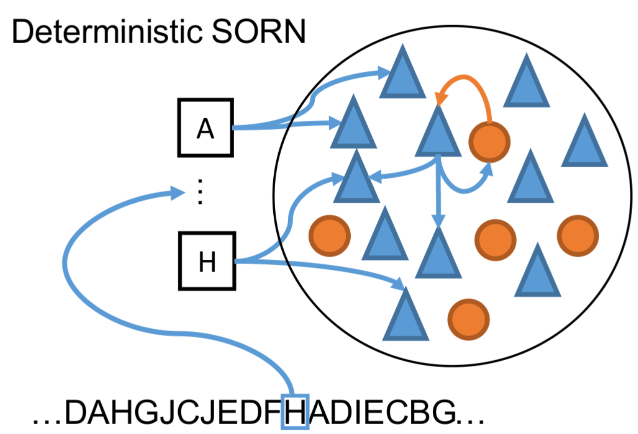


Hartmann et al, PLoS Comput Biol 2015

deterministic network



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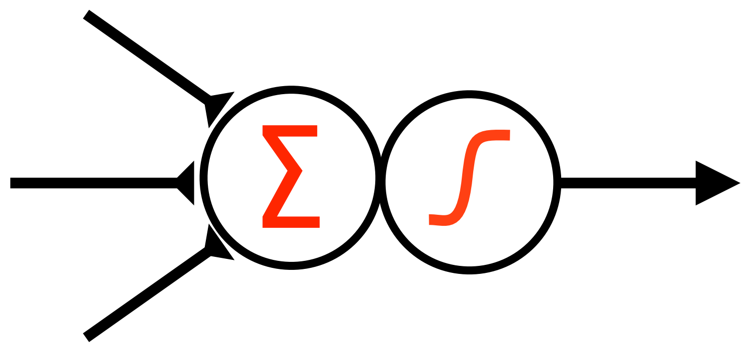

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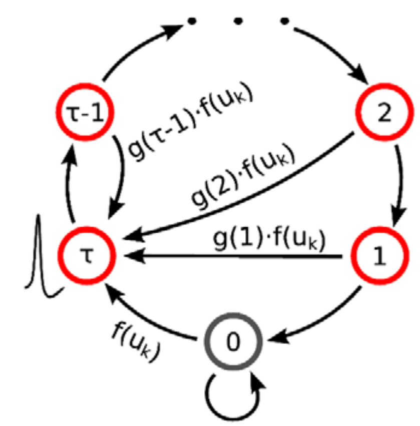

Hartmann et al, PLoS Comput Biol 2015

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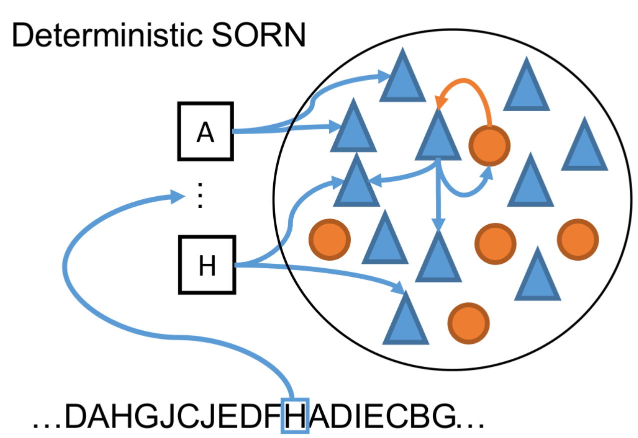
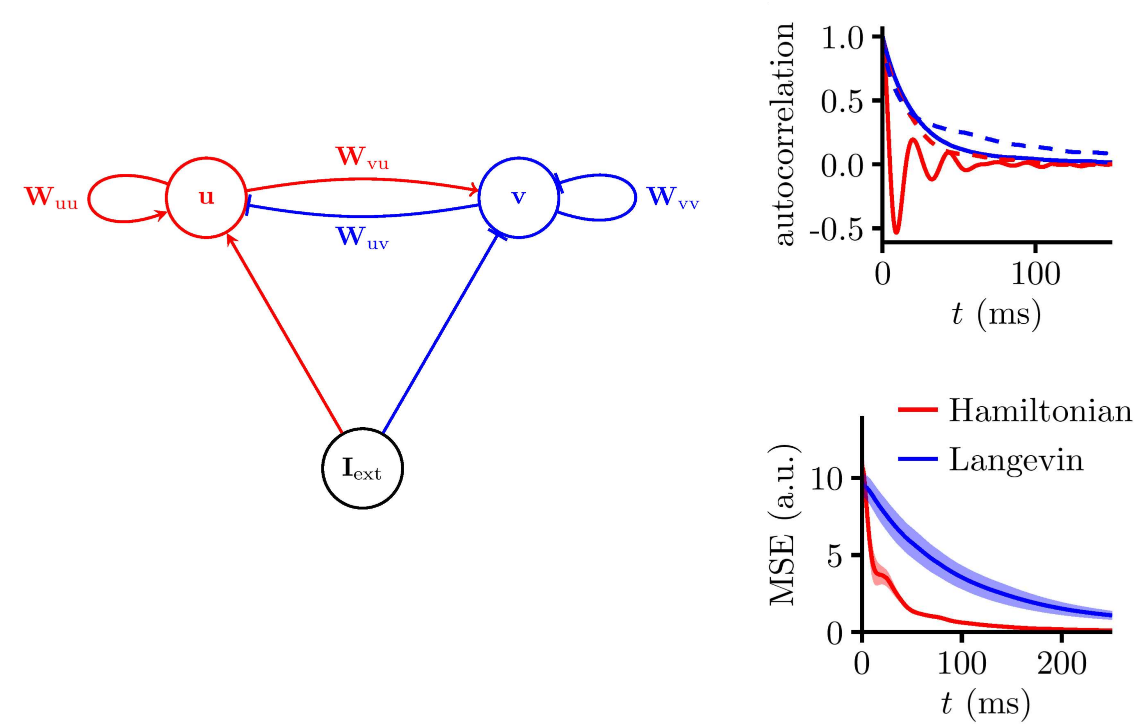


Hennequin et al, NIPS 2014
Aitchison & Lengyel, PLoS Comput Biol 2015

fast (Hamiltonian) sampling in E/I networks



NEURAL CIRCUIT DYNAMICS

see talks later today by

Guillaume Hennequin

Rodrigo Echeveste

Laurence Aitchison

Cristina Savin

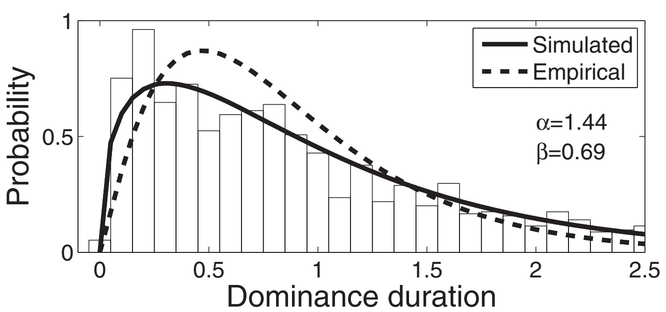
Jean-Pascal Pfister

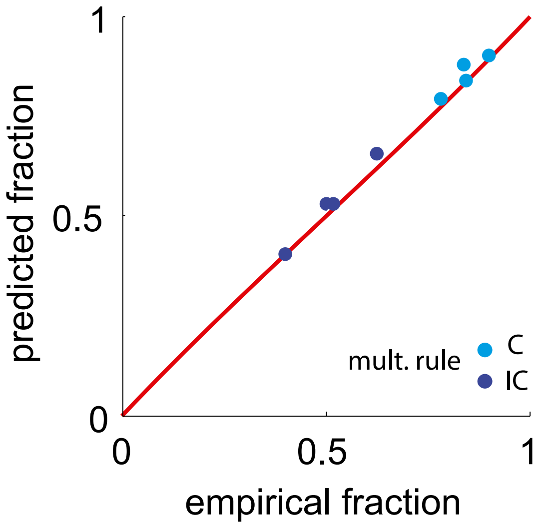
BEHAVIORAL DYNAMICS

BEHAVIORAL DYNAMICS

Sundareswara & Schrater, J Vis 2008
Gershman et al, NIPS 2009; Neural Comput 2012
Moreno-Bote et al, PNAS 2011

perceptual multistability





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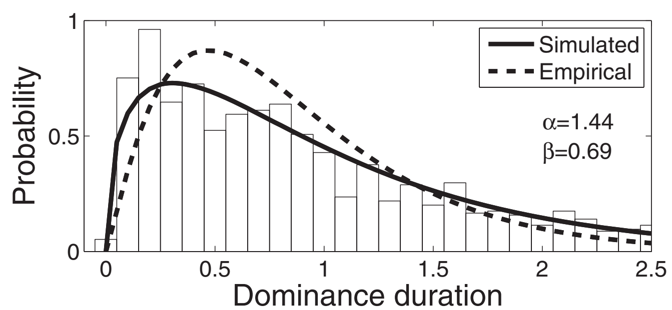
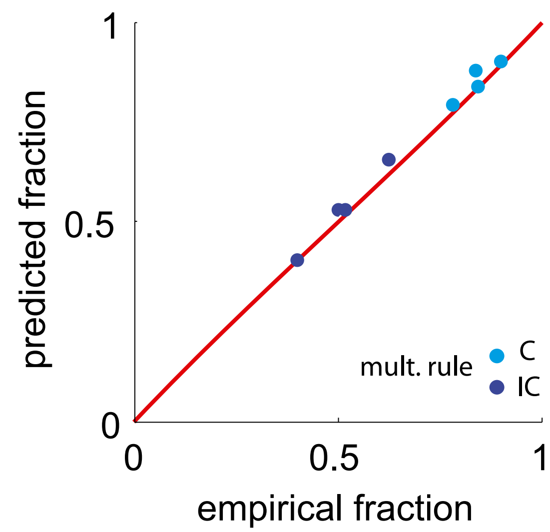
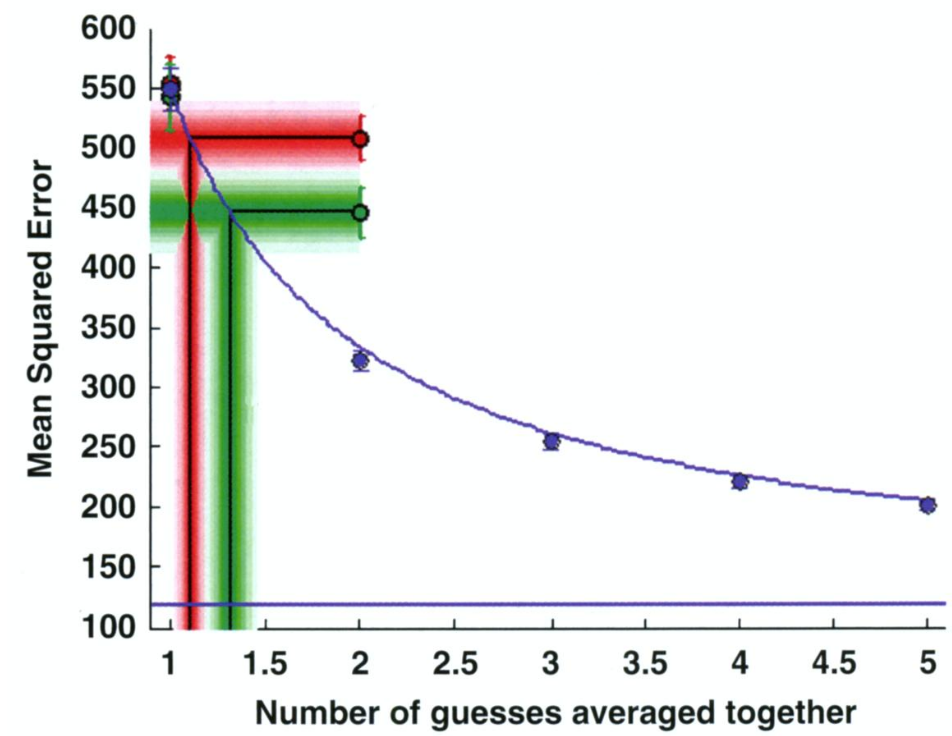




Vul & Pashler, Psych Sci 2008
Vul et al, Cog Sci 2014

idiosyncrasies in decision making



BEHAVIORAL DYNAMICS

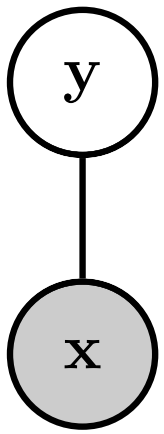
see talk tomorrow by

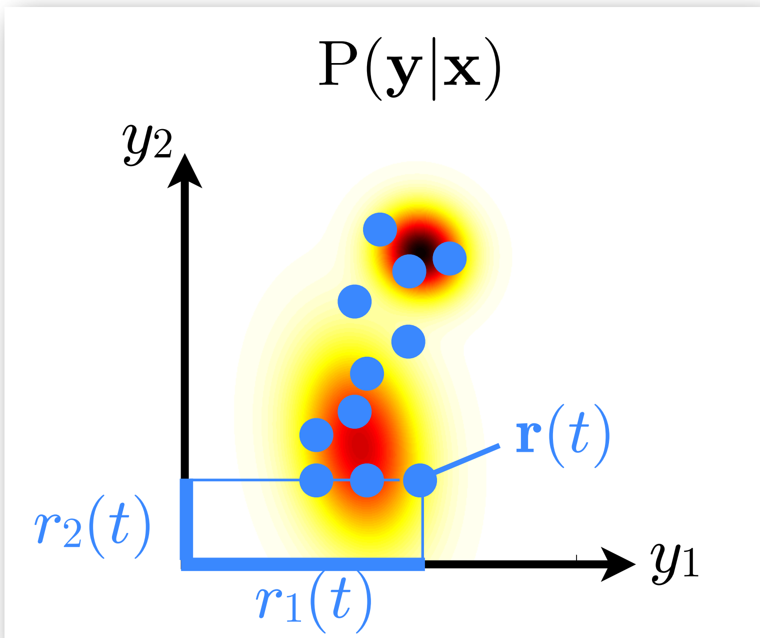
Adam Sanborn

A SIMPLE TAXONOMY OF PROBABILISTIC REPRESENTATIONS

probability distribution

spatio-temporal neural activity patterns

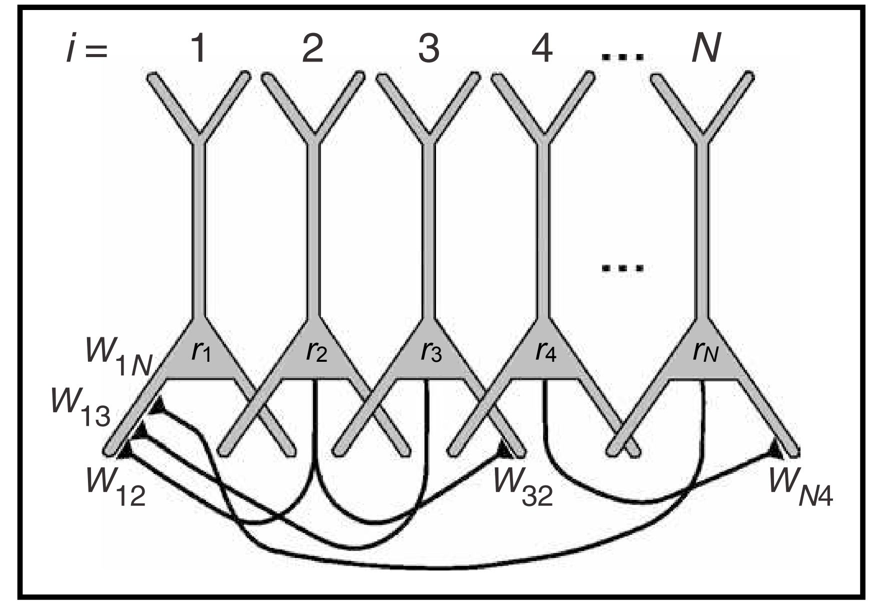


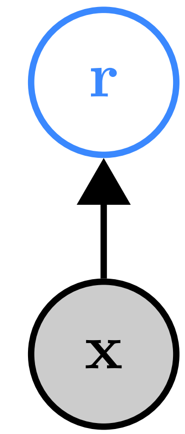


sampling-based

$\mathbf{r} \sim \mathrm{P}(\mathbf{y} = \mathbf{r}|\mathbf{x})$

$\mathbf{r}(t,\mathbf{x})$

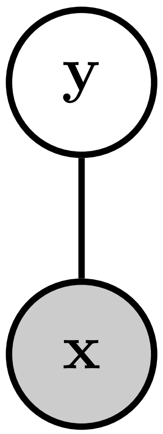


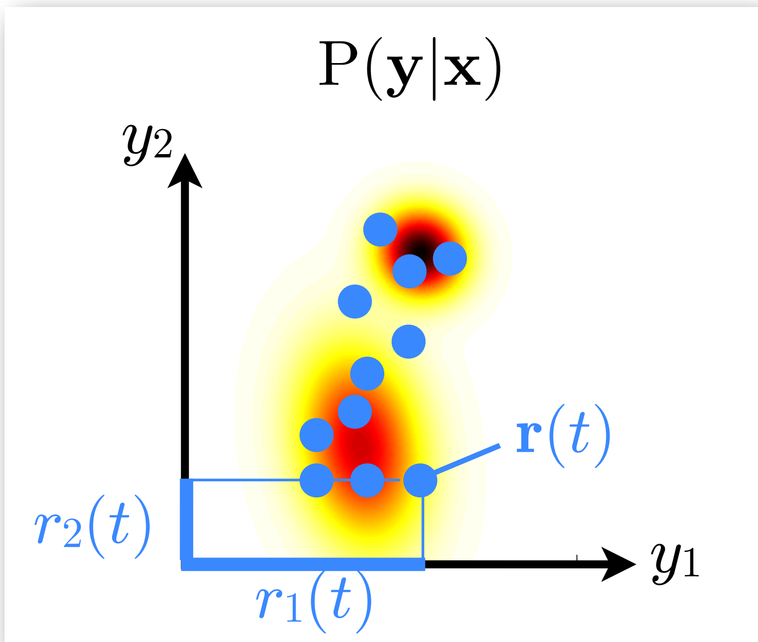


A SIMPLE TAXONOMY OF PROBABILISTIC REPRESENTATIONS

probability distribution

spatio-temporal neural activity patterns

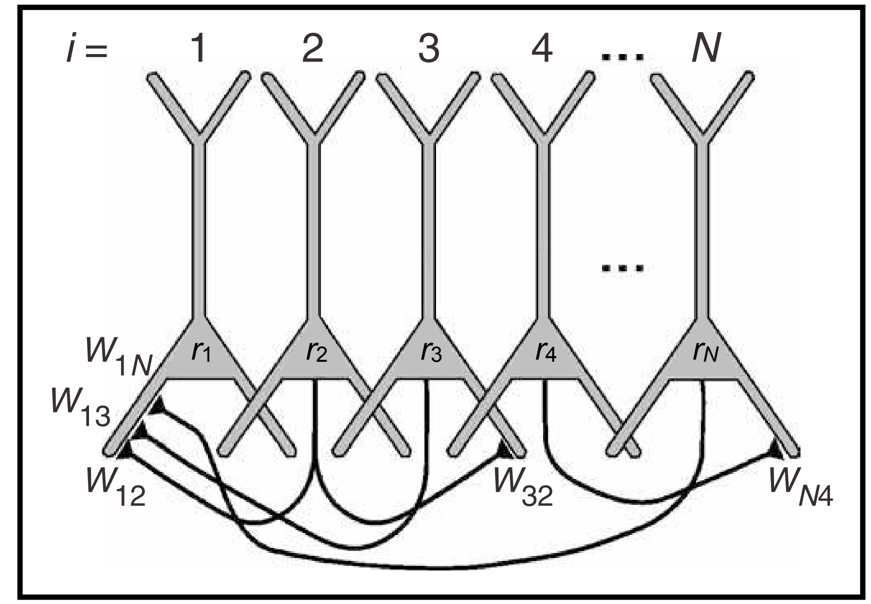


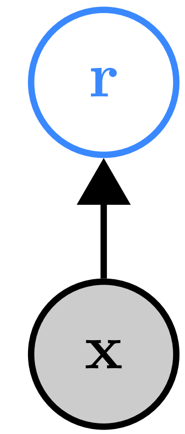


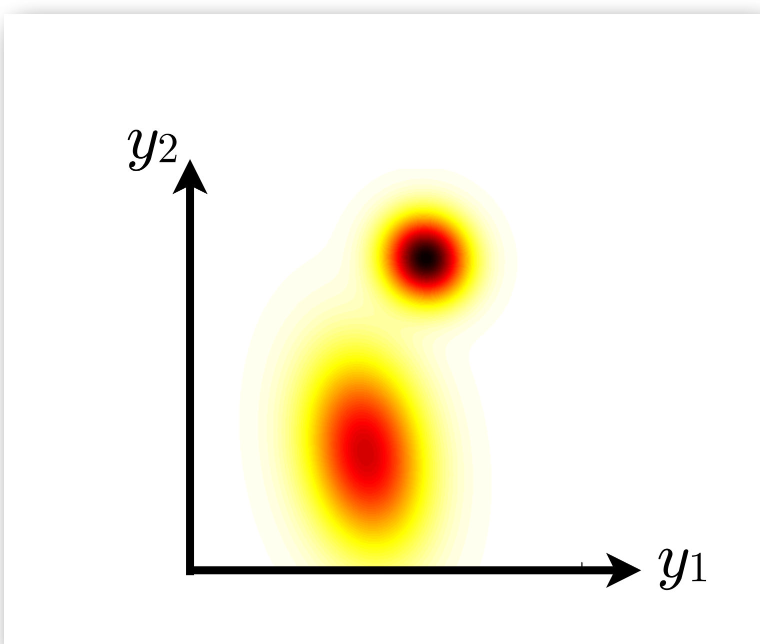
sampling-based

$$\mathbf{r} \sim \mathrm{P}(\mathbf{y} = \mathbf{r}|\mathbf{x})$$

$\mathbf{r}(t,\mathbf{x})$



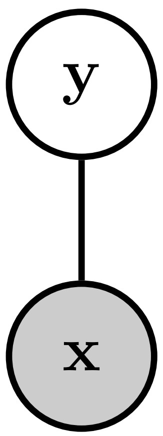


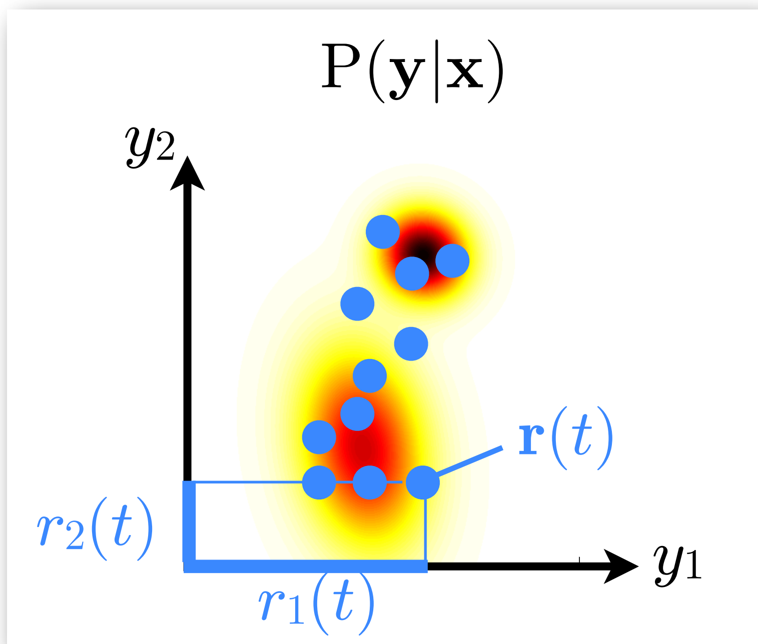


A SIMPLE TAXONOMY OF PROBABILISTIC REPRESENTATIONS

probability distribution

spatio-temporal neural activity patterns





sampling-based

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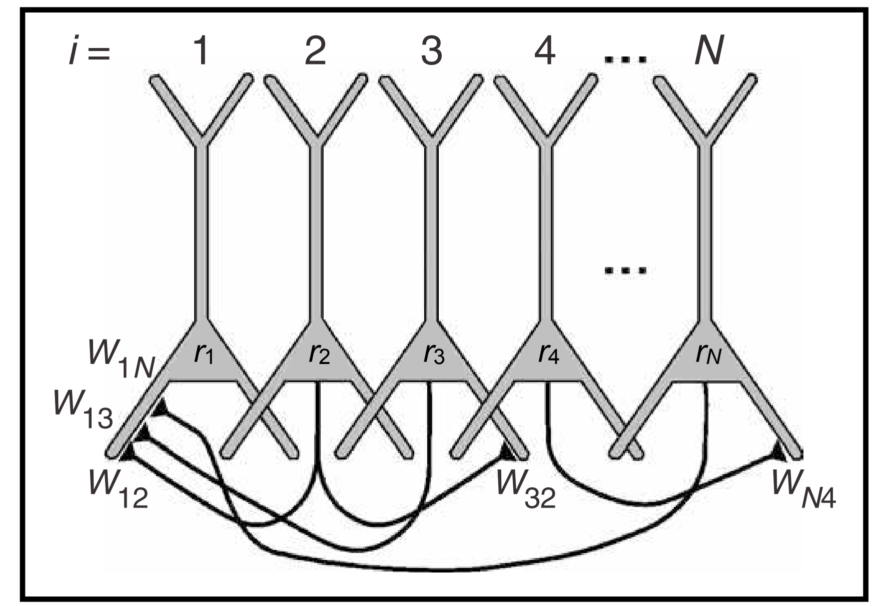
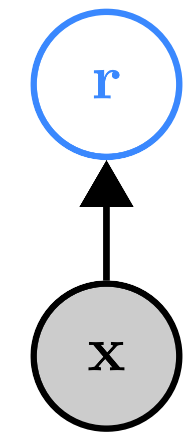
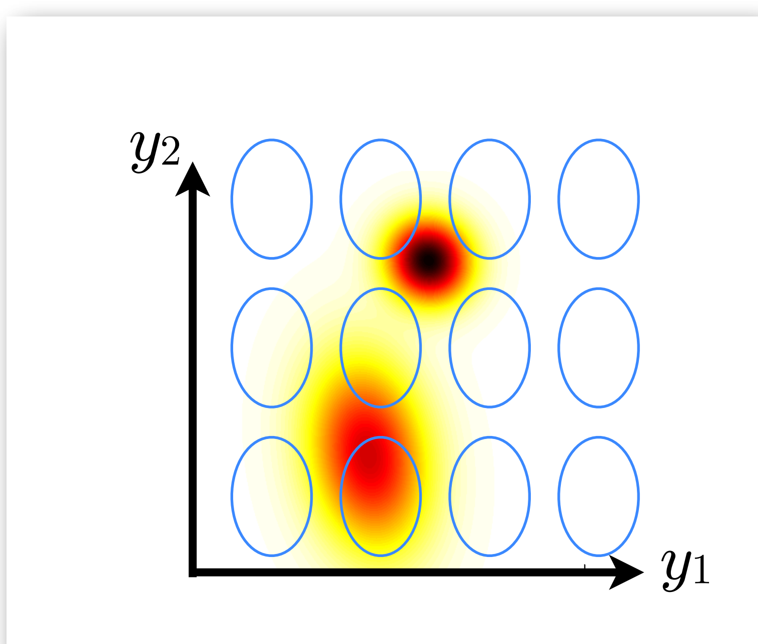
$\mathbf{r}(t,\mathbf{x})$







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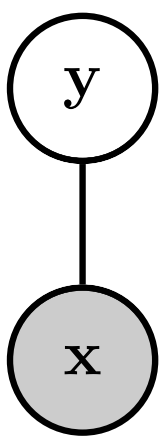

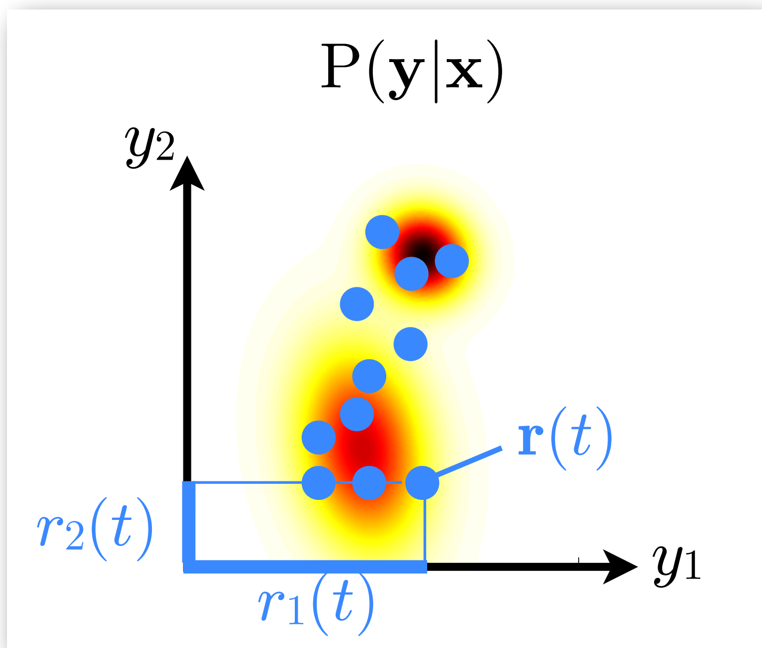

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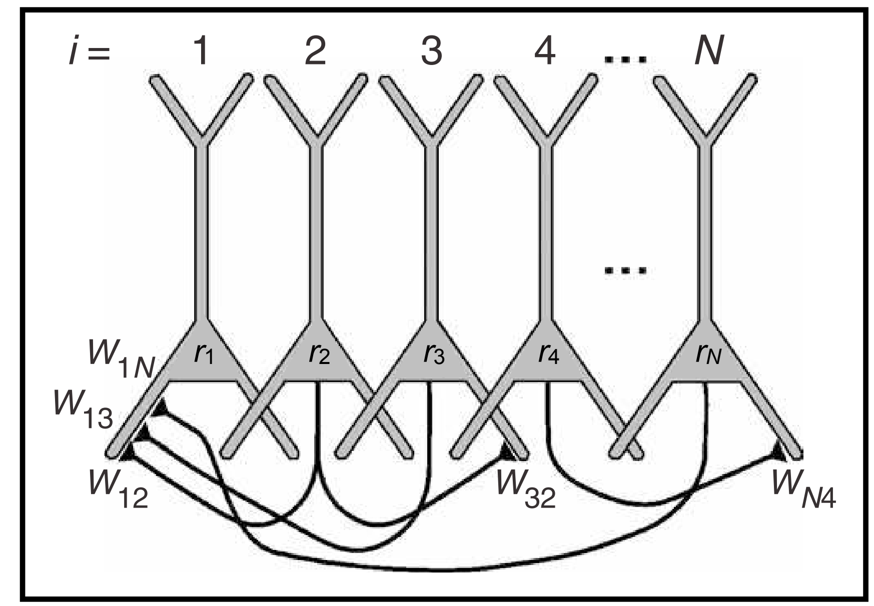

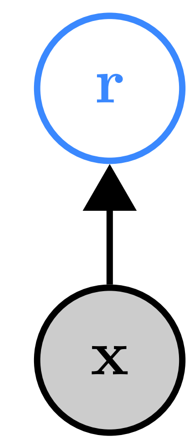

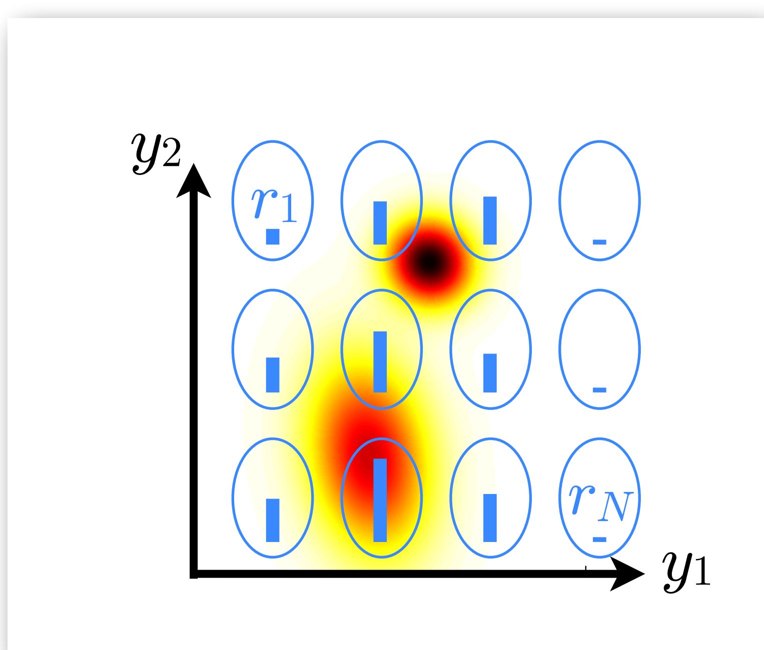

A SIMPLE TAXONOMY OF PROBABILISTIC REPRESENTATIONS

probability distribution

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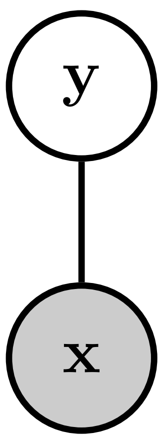

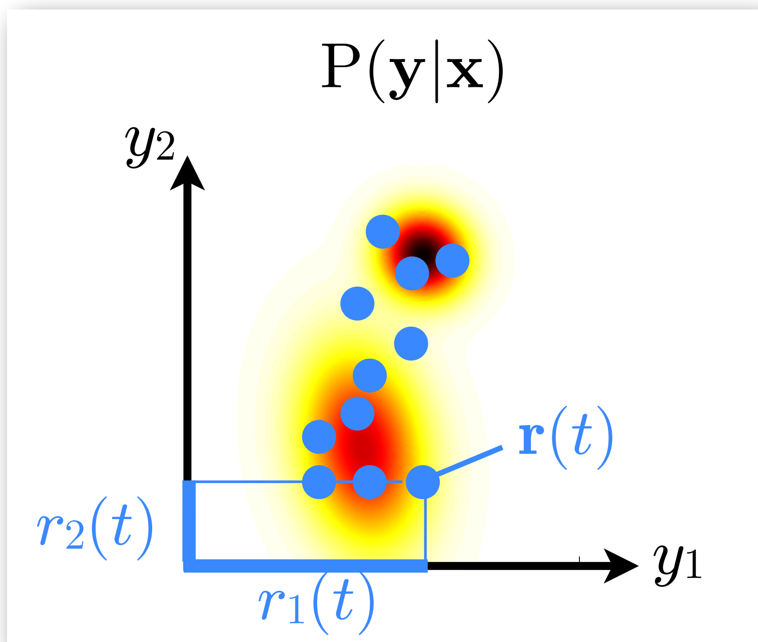

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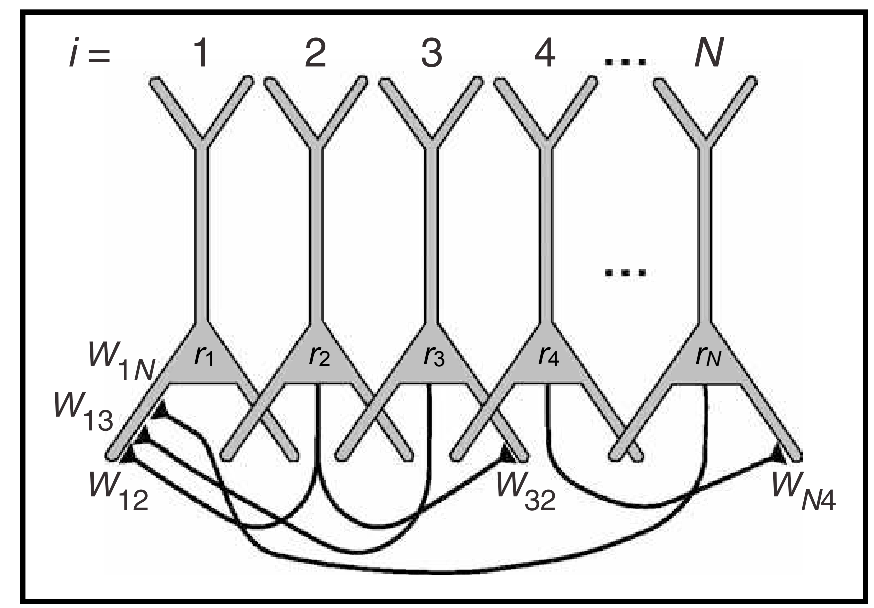

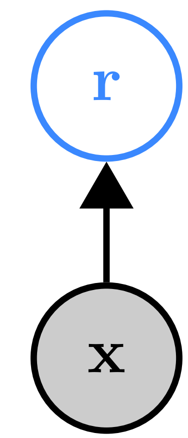

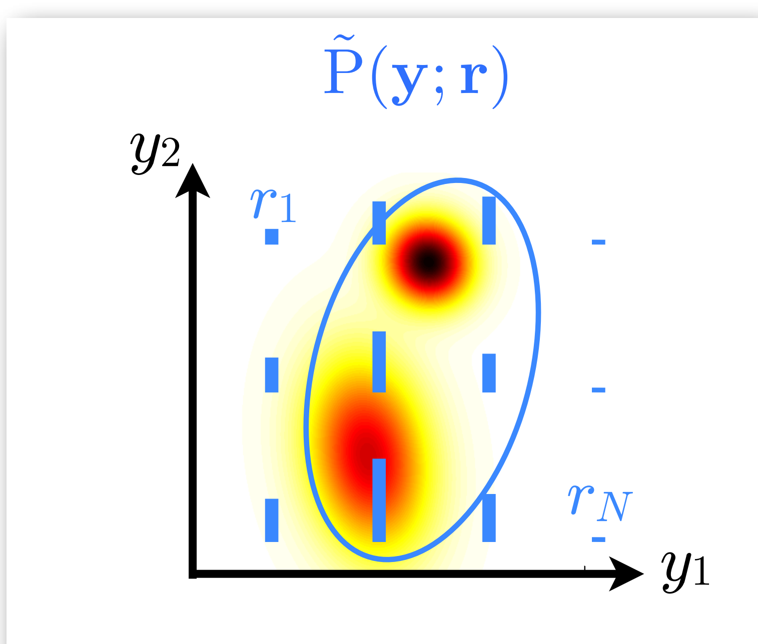

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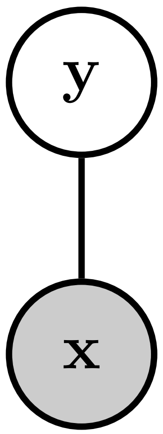

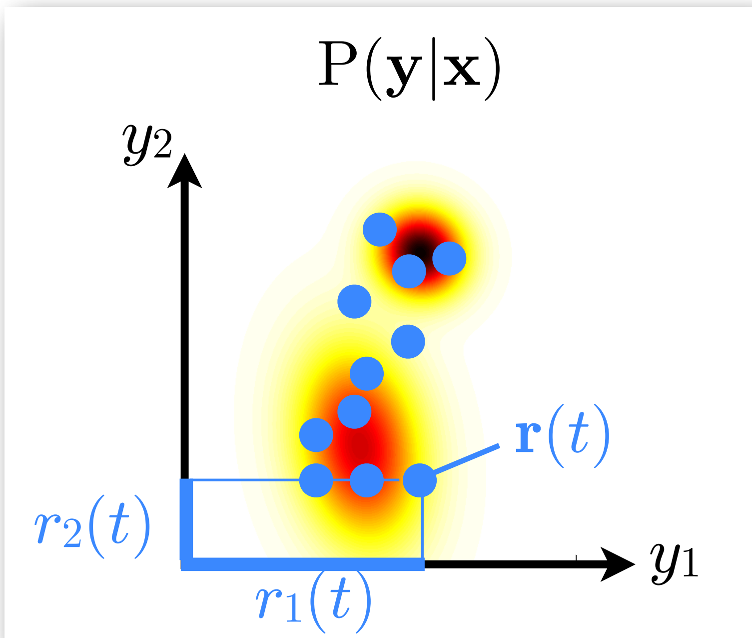

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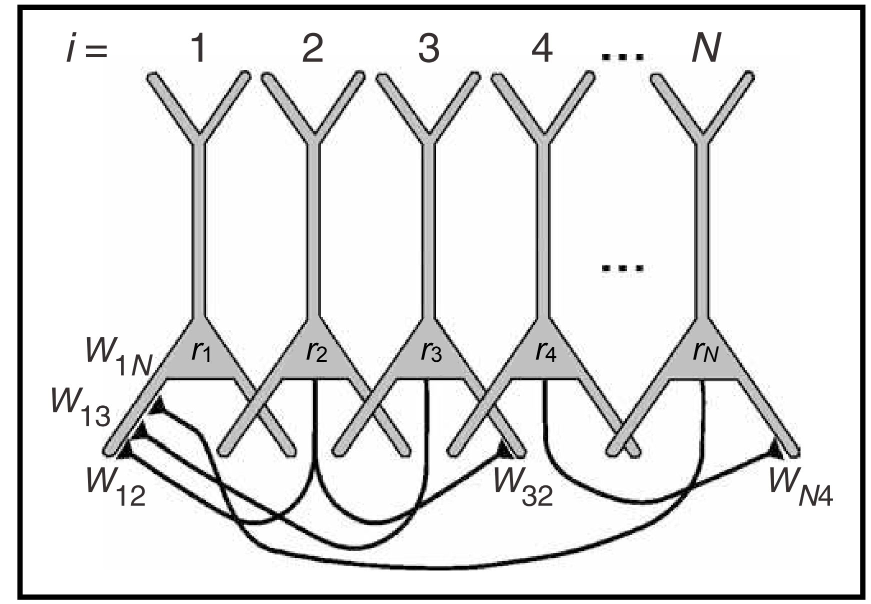
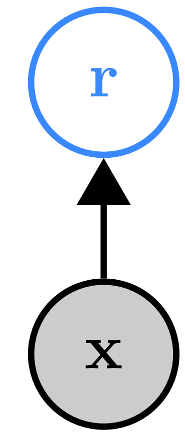
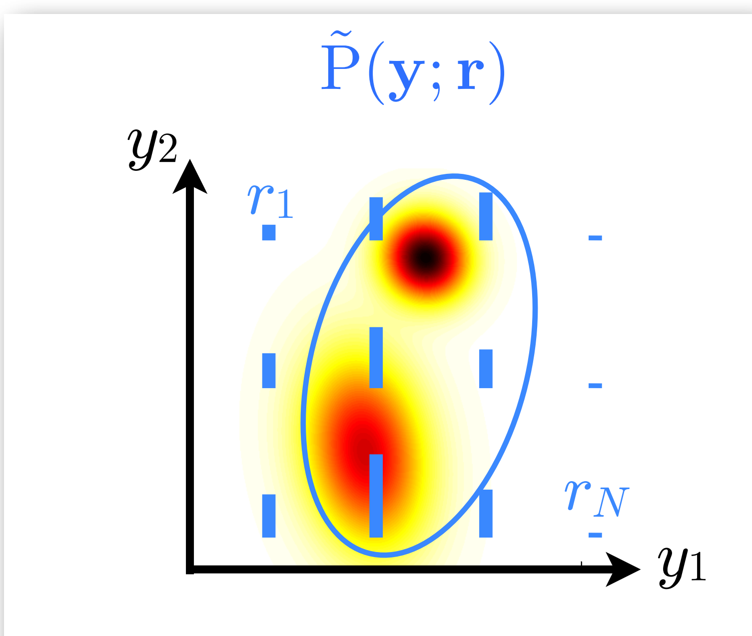






parametric

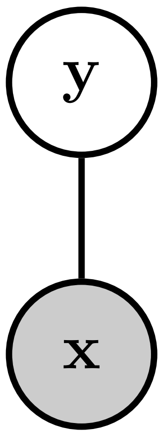
$$\mathbf{r} = \operatorname{argmin} \mathrm{KL}\left[\tilde{\mathrm{P}}(\mathbf{y};\mathbf{r}) \parallel \mathrm{P}(\mathbf{y}|\mathbf{x})\right]$$ instantaneous

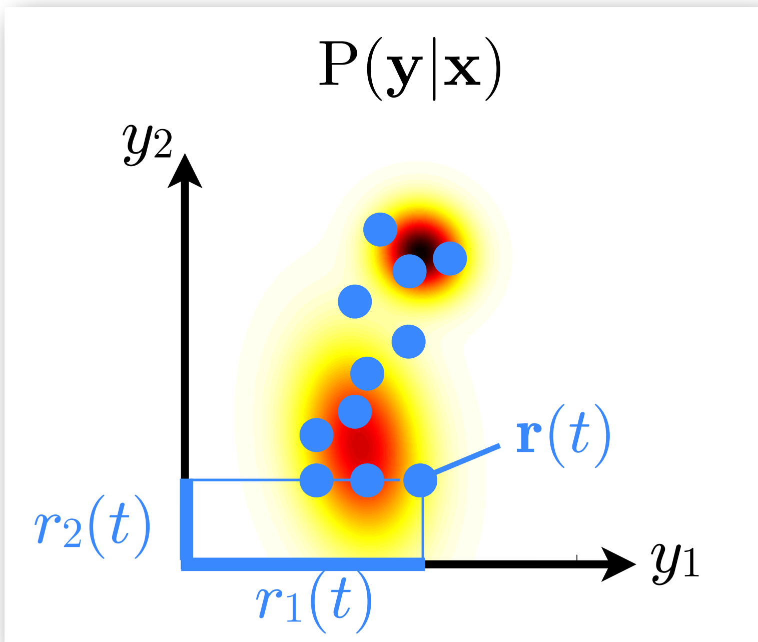
$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{r} = -\nabla_{\mathbf{r}} \mathrm{KL}\left[\tilde{\mathrm{P}}(\mathbf{y};\mathbf{r}) \parallel \mathrm{P}(\mathbf{y}|\mathbf{x})\right]$$ iterative

A SIMPLE TAXONOMY OF PROBABILISTIC REPRESENTATIONS

probability distribution

spatio-temporal neural activity patterns





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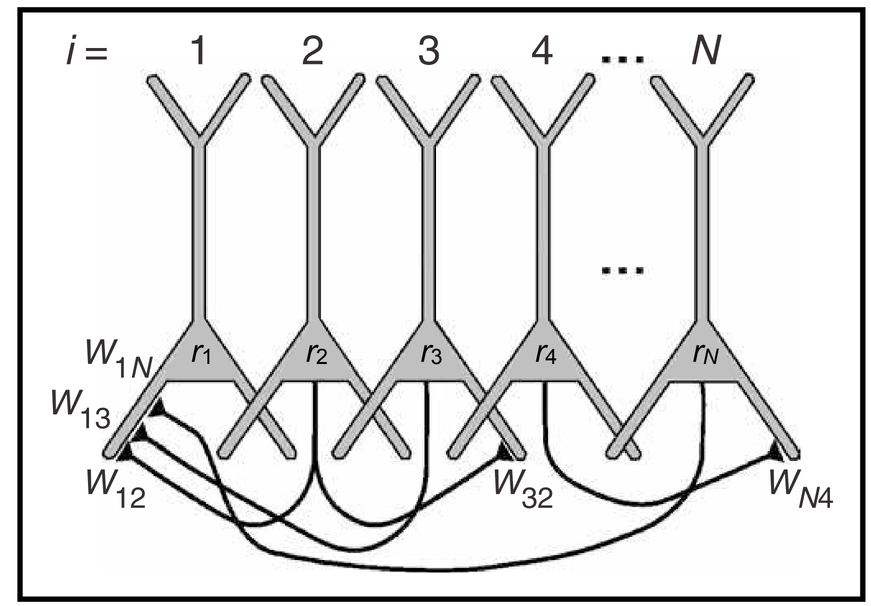

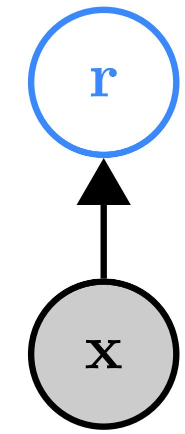

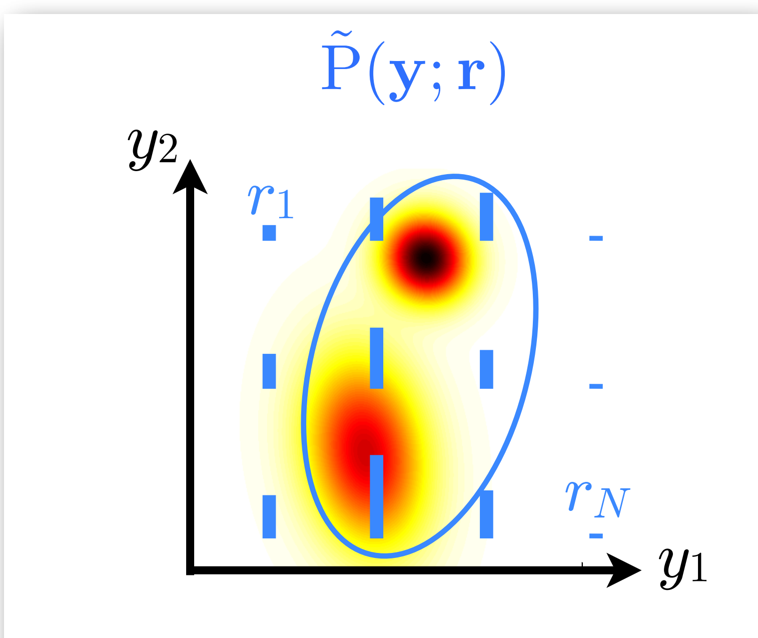

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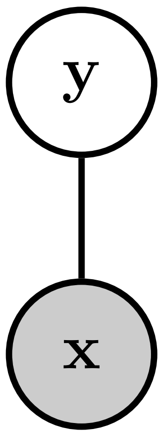
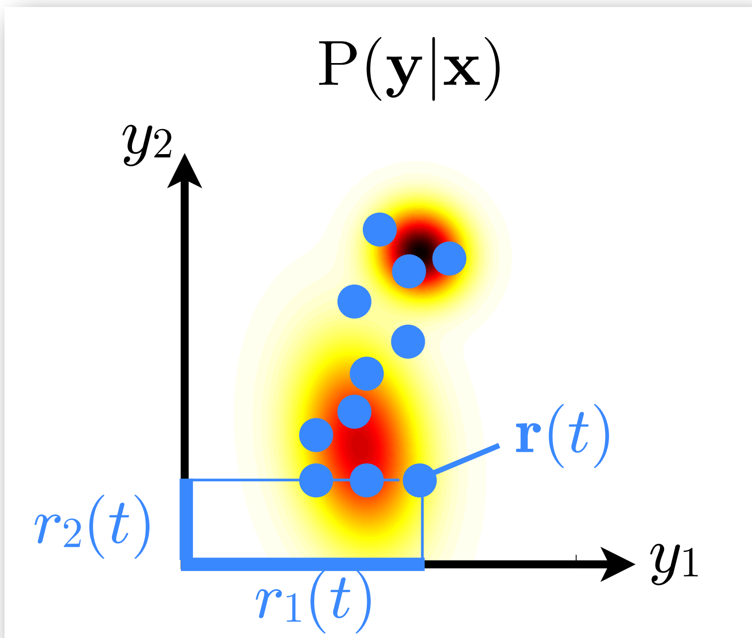
probabilistic population codes
(product of experts)

A SIMPLE TAXONOMY OF PROBABILISTIC REPRESENTATIONS

probability distribution | spatio-temporal neural activity patterns





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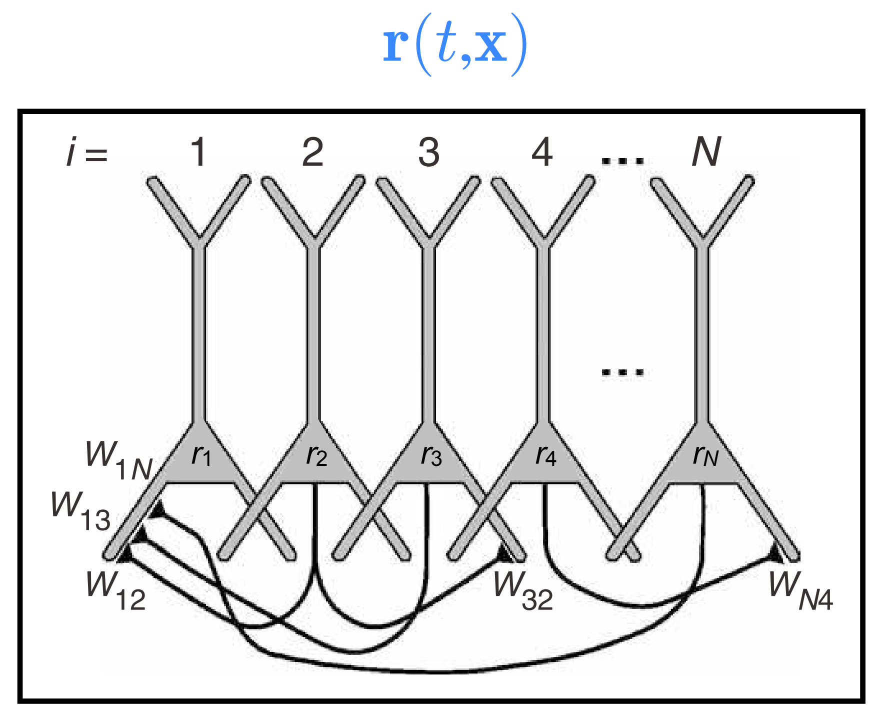
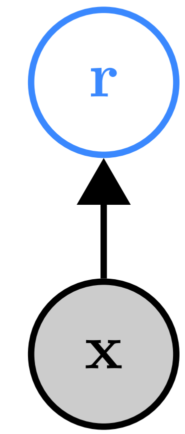
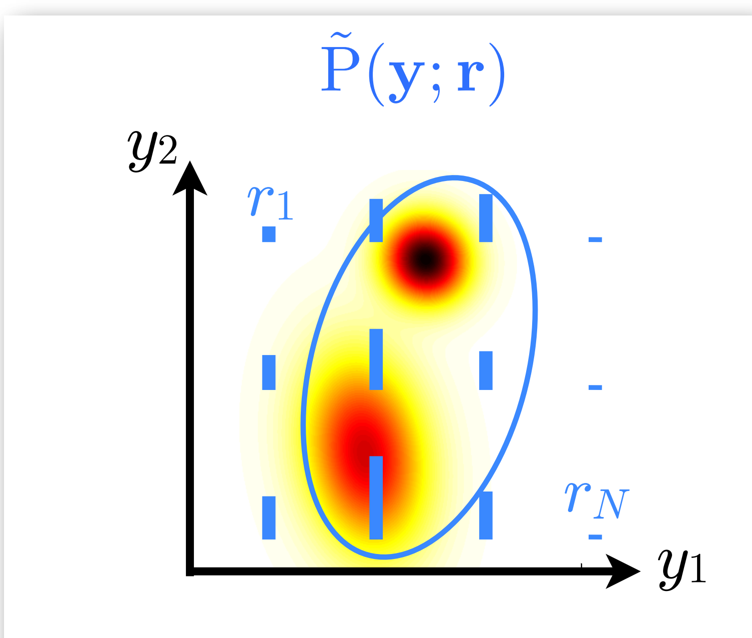
$$\mathbf{r} \sim \mathrm{P}(\mathbf{y} = \mathbf{r}|\mathbf{x})$$





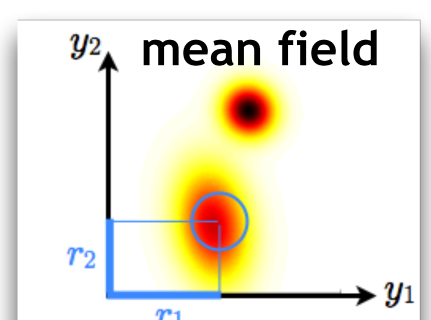


parametric

$$\mathbf{r} = \mathrm{argmin}\ \mathrm{KL}\left[\tilde{\mathrm{P}}(\mathbf{y};\mathbf{r}) \parallel \mathrm{P}(\mathbf{y}|\mathbf{x})\right]$$ instantaneous

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathbf{r} = -\nabla_{\mathbf{r}}\ \mathrm{KL}\left[\tilde{\mathrm{P}}(\mathbf{y};\mathbf{r}) \parallel \mathrm{P}(\mathbf{y}|\mathbf{x})\right]$$ iterative

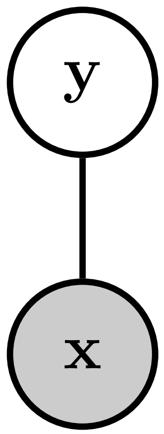
probabilistic population codes
(product of experts)

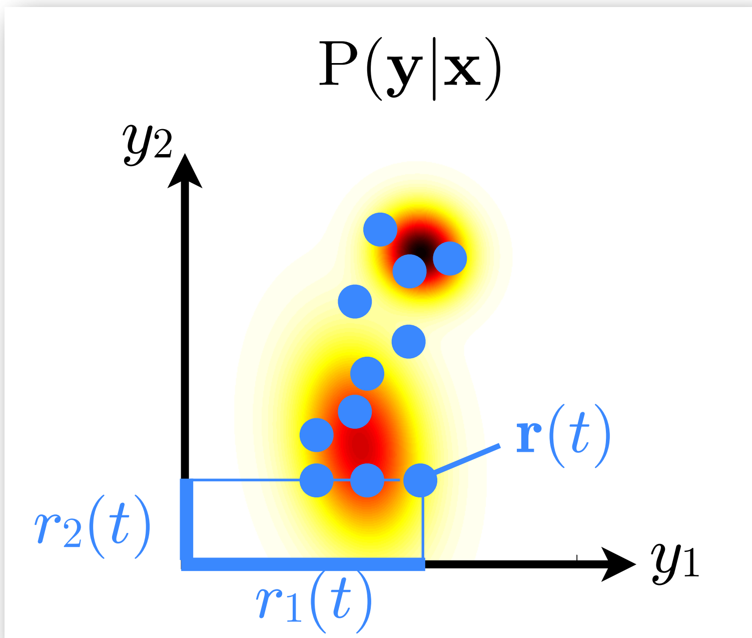


A SIMPLE TAXONOMY OF PROBABILISTIC REPRESENTATIONS

probability distribution

spatio-temporal neural activity patterns

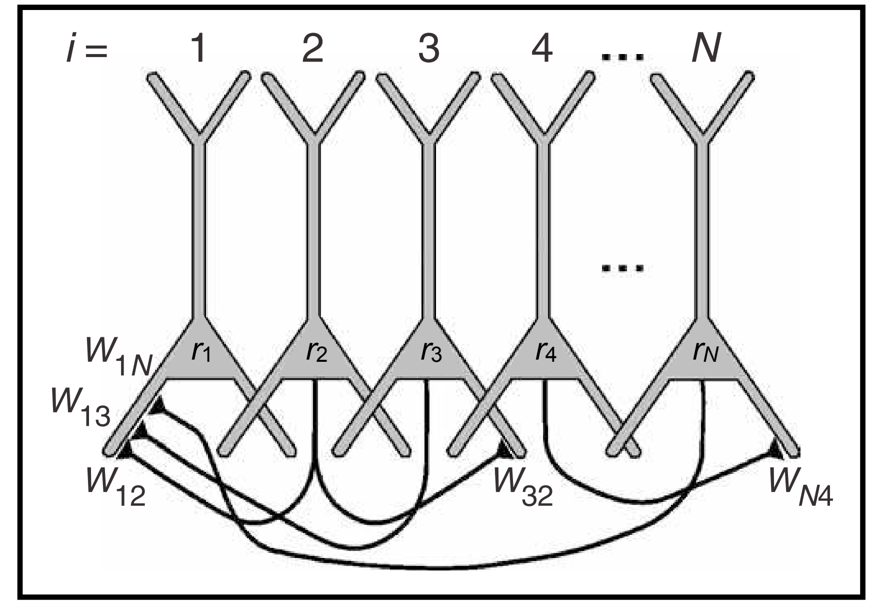


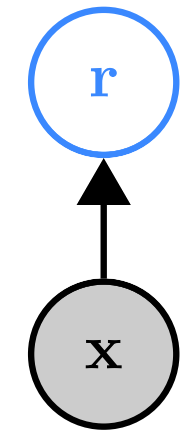


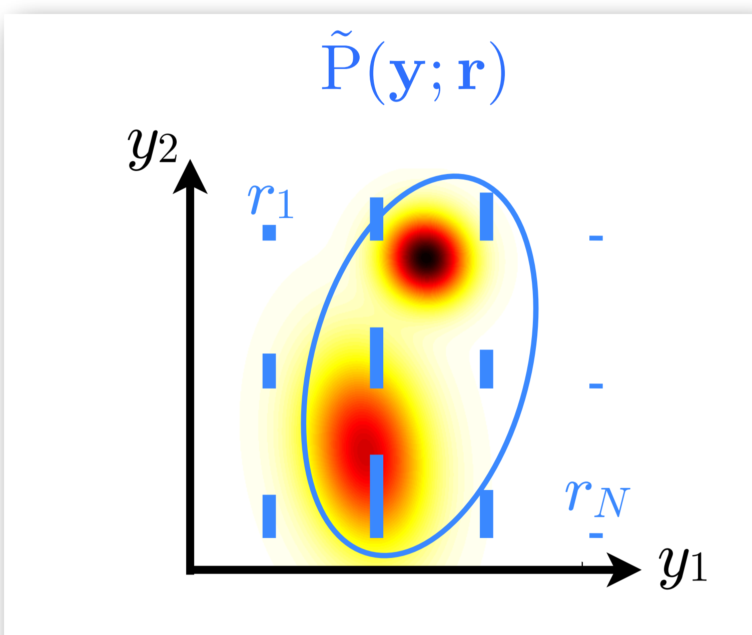
sampling-based

$\mathbf{r} \sim \mathrm{P}(\mathbf{y} = \mathbf{r}|\mathbf{x})$

$\mathbf{r}(t,\mathbf{x})$







parametric

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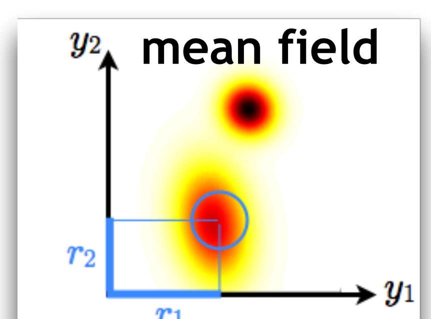
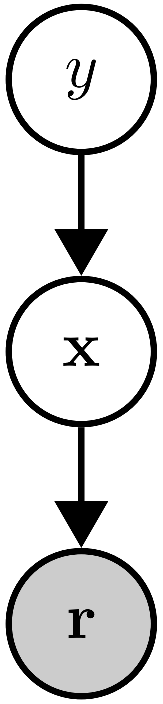
→ **probabilistic population codes (product of experts)**

→ 

→ **predictive coding**

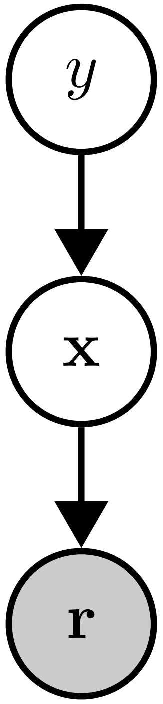
PROBABILISTIC POPULATION CODES

PROBABILISTIC POPULATION CODES



neural responses $\mathbf{r}$ to stimuli y are variable: $\mathrm{P}(\mathbf{r}|y) = \int \mathrm{P}(\mathbf{r}|\mathbf{x})\ \mathrm{P}(\mathbf{x}|y)\ d\mathbf{x}$

PROBABILISTIC POPULATION CODES



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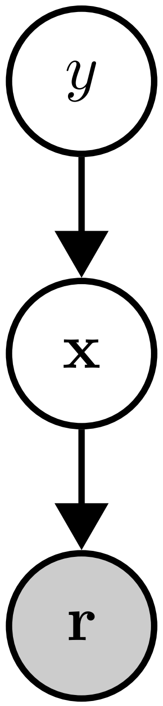
decoding neural responses yields a distribution: $\mathrm{P}(y|\mathbf{r}) \propto \mathrm{P}(\mathbf{r}|y)\ \mathrm{P}(y)$

PROBABILISTIC POPULATION CODES



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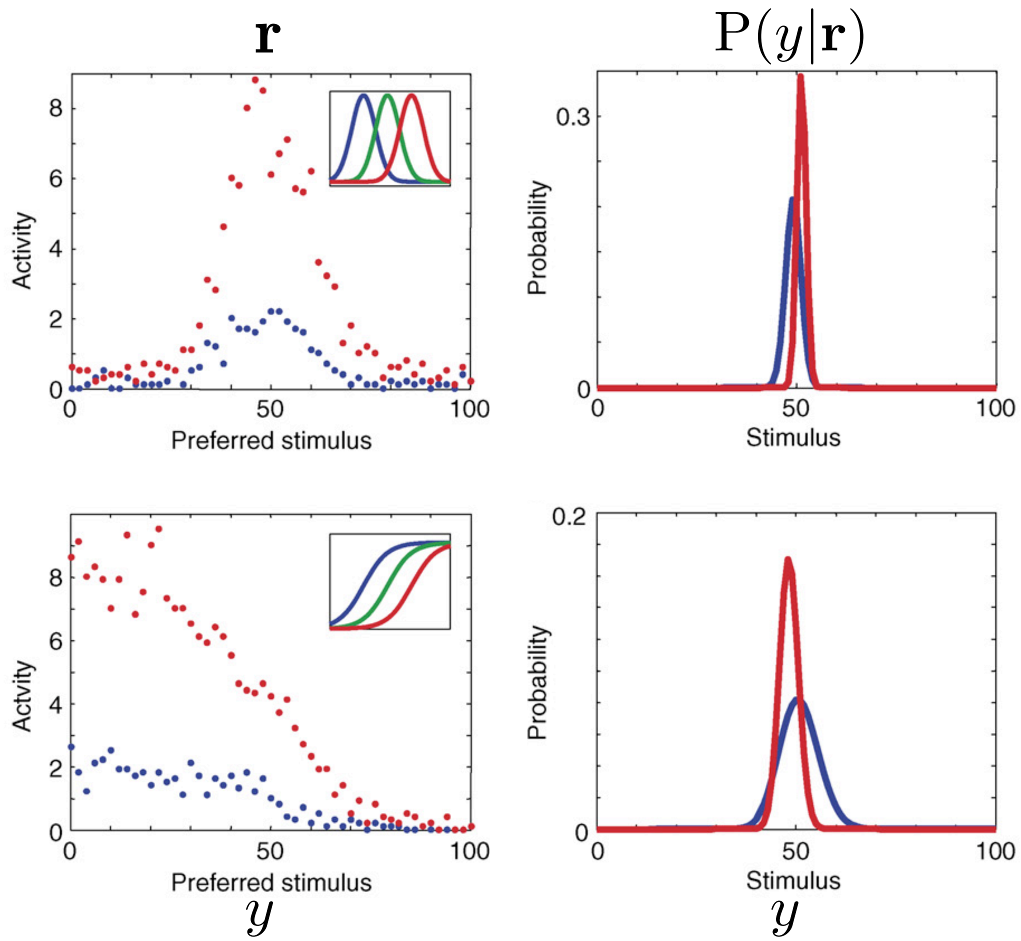
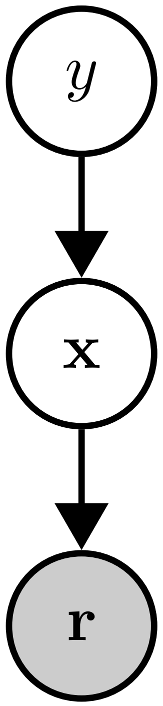


PROBABILISTIC POPULATION CODES



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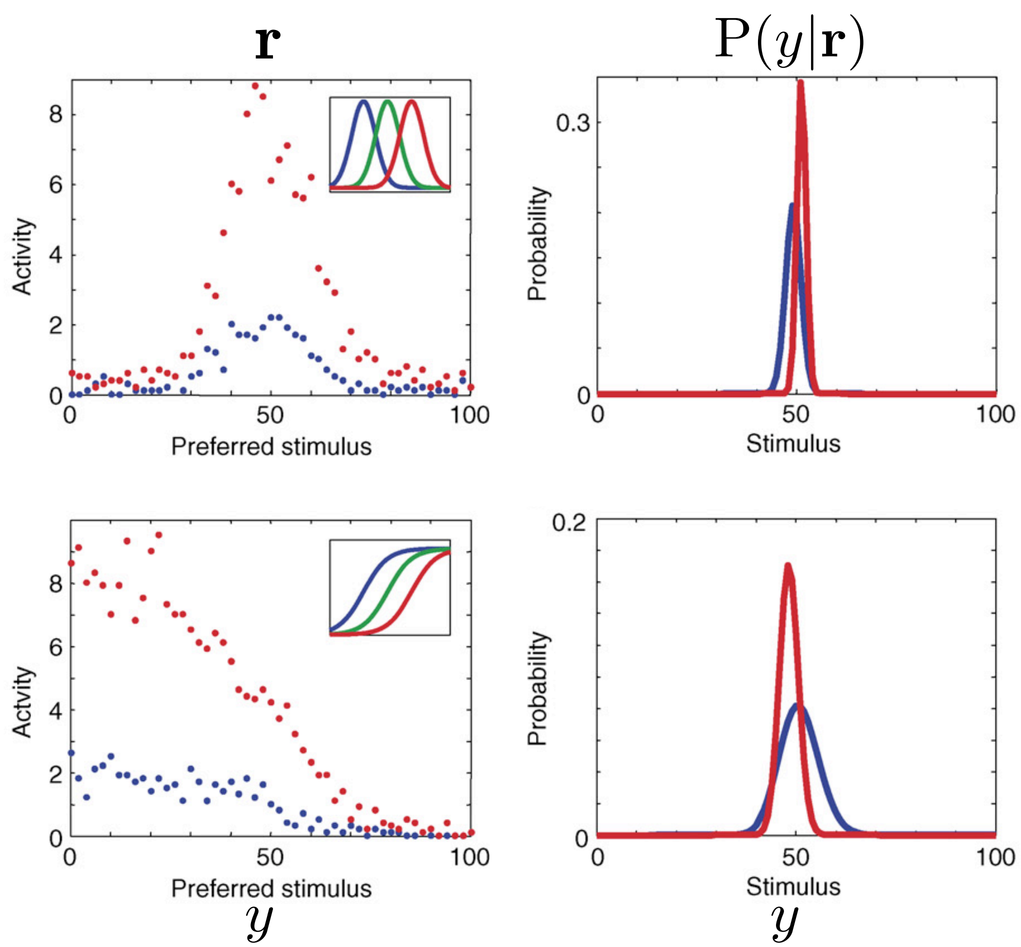
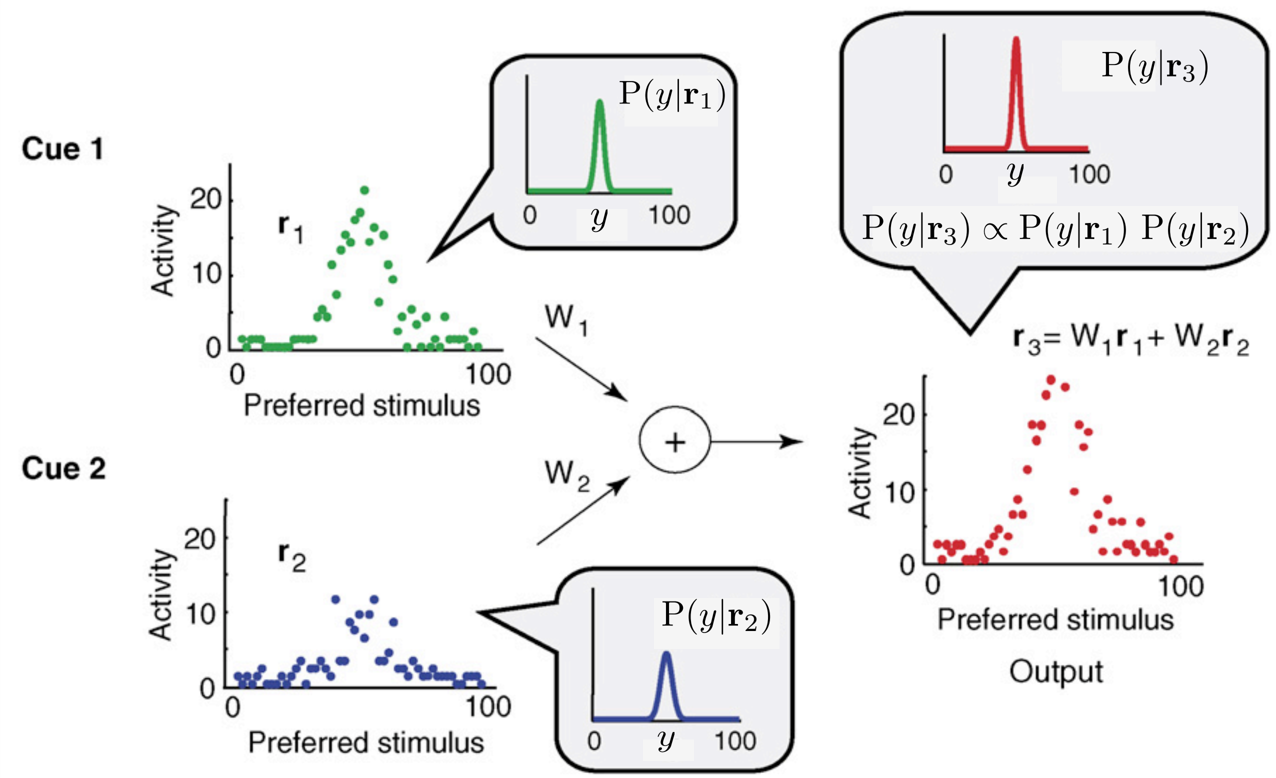


if $\mathrm{P}(\mathbf{r}|y)$ is 'Poisson-like'

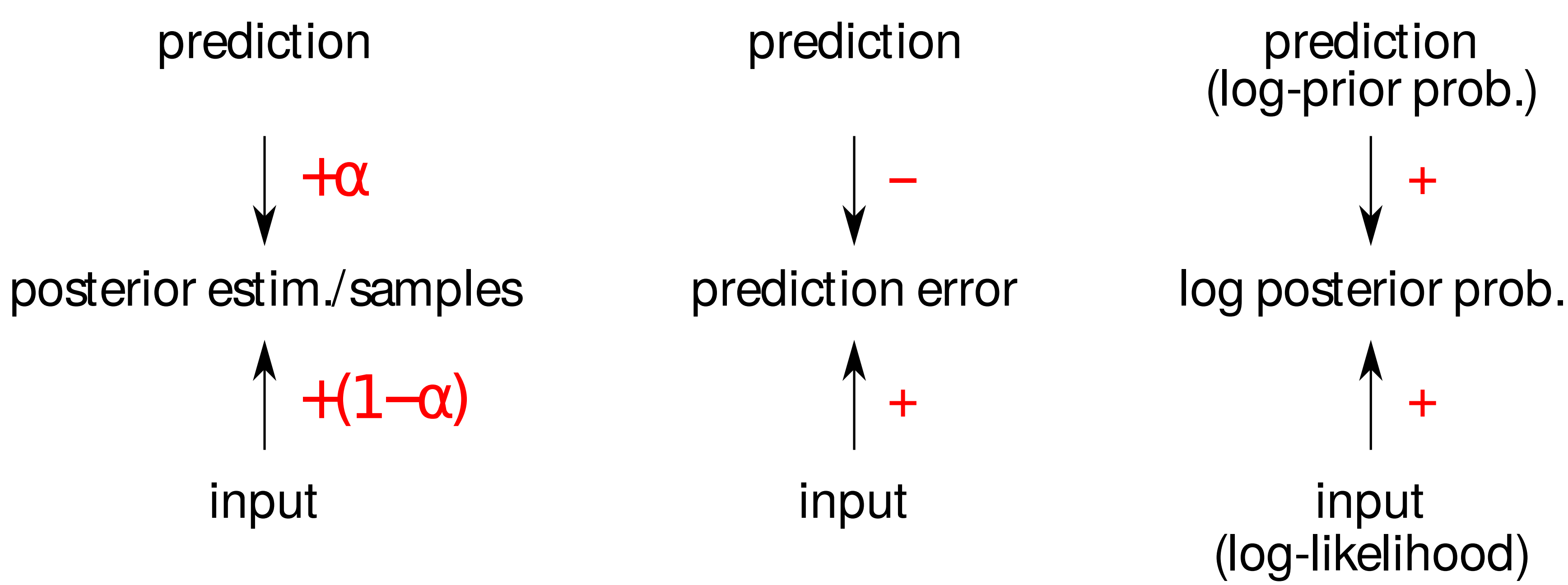
cue integration by summation



Ma & al, 2008

NEURAL ARITHMETICS

sampling *predictive coding* *PPC*



Aitchison & Lengyel, Curr Opin Neurobiol, in press

NEURAL REPRESENTATIONS OF UNCERTAINTY
computational properties

NEURAL REPRESENTATIONS OF UNCERTAINTY
computational properties

	sampling	**mean-field**	**prob. pop. code**
neurons represent	variables	parameters	parameters

NEURAL REPRESENTATIONS OF UNCERTAINTY
computational properties

	sampling	**mean-field**	**prob. pop. code**
neurons represent	variables	parameters	parameters
neurons / variable	1	1	many (~100—1000)

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computational properties

	sampling	**mean-field**	**prob. pop. code**	
neurons represent	variables	parameters	parameters	
neurons / variable	1	1	many (~100—1000)	too many?

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distributions are represented	by iterative sampling	by iterative dynamics / instantaneously	instantaneously	

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distributions are represented	by iterative sampling	by iterative dynamics / instantaneously	instantaneously	too slow!?
inference of dynamical variables	?	✓	✓	

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neurons / variable	1	1	many (~100—1000)	too many?
distributions are represented	by iterative sampling	by iterative dynamics / instantaneously	instantaneously	too slow!?
inference of dynamical variables	?	✓	✓	
correlations (limiting factor)	✓ (time)	✗	✓ (neurons)	

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	sampling	mean-field	prob. pop. code	
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distributions are represented	by iterative sampling	by iterative dynamics / instantaneously	instantaneously	too slow!?
inference of dynamical variables	?	✓	✓	
correlations (limiting factor)	✓ (time)	✗	✓ (neurons)	
cue combination	✓	✓	✓	

NEURAL REPRESENTATIONS OF UNCERTAINTY
computational properties

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neurons represent	variables	parameters	parameters	
neurons / variable	1	1	many (~100—1000)	too many?
distributions are represented	by iterative sampling	by iterative dynamics / instantaneously	instantaneously	too slow!?
inference of dynamical variables	?	✓	✓	
correlations (limiting factor)	✓ (time)	✗	✓ (neurons)	
cue combination	✓	✓	✓	
marginalisation	✓	✓	✓?	

NEURAL REPRESENTATIONS OF UNCERTAINTY
computational properties

	sampling	mean-field	prob. pop. code	
neurons represent	variables	parameters	parameters	
neurons / variable	1	1	many (~100—1000)	too many?
distributions are represented	by iterative sampling	by iterative dynamics / instantaneously	instantaneously	too slow!?
inference of dynamical variables	?	✓	✓	
correlations (limiting factor)	✓ (time)	✗	✓ (neurons)	
cue combination	✓	✓	✓	
marginalisation	✓	✓	✓?	
dynamics	stochastic	deterministic	deterministic	

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computational properties

	sampling	**mean-field**	**prob. pop. code**	
neurons represent	variables	parameters	parameters	
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distributions are represented	by iterative sampling	by iterative dynamics / instantaneously	instantaneously	too slow!?
inference of dynamical variables	?	✓	✓	
correlations (limiting factor)	✓ (time)	✗	✓ (neurons)	
cue combination	✓	✓	✓	
marginalisation	✓	✓	✓?	
dynamics	stochastic	deterministic	deterministic	robustness?

NEURAL REPRESENTATIONS OF UNCERTAINTY
computational properties

	sampling	mean-field	prob. pop. code	
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neurons / variable	1	1	many (~100—1000)	too many?
distributions are represented	by iterative sampling	~~by iterative dynamics /~~ instantaneously	instantaneously	too slow!?
inference of dynamical variables	?	✓	✓	
correlations (limiting factor)	✓ (time)	✗	✓ (neurons)	
cue combination	✓	✓	✓	
marginalisation	✓	✓	✓?	
dynamics	stochastic	deterministic	deterministic	robustness?
neural variability for computation	useful	harmful	harmful	

NEURAL REPRESENTATIONS OF UNCERTAINTY
computational properties

	sampling	**mean-field**	**prob. pop. code**	
neurons represent	variables	parameters	parameters	
neurons / variable	1	1	many (~100—1000)	too many?
distributions are represented	by iterative sampling	~~by iterative dynamics /~~ instantaneously	instantaneously	too slow!?
inference of dynamical variables	?	✓	✓	
correlations (limiting factor)	✓ (time)	✗	✓ (neurons)	
cue combination	✓	✓	✓	
marginalisation	✓	✓	✓?	
dynamics	stochastic	deterministic	deterministic	robustness?
neural variability for computation	useful	harmful	harmful	
learning	✓	✓?	?	

NEURAL REPRESENTATIONS OF UNCERTAINTY
accounting for neural data

NEURAL REPRESENTATIONS OF UNCERTAINTY
accounting for neural data

	sampling	mean-field	prob. pop. code
variability	✓	✗	✓

NEURAL REPRESENTATIONS OF UNCERTAINTY
accounting for neural data

	sampling	**mean-field**	**prob. pop. code**
variability	✓	✗	✓
noise correlations	✓	✗	✓

NEURAL REPRESENTATIONS OF UNCERTAINTY
accounting for neural data

	sampling	mean-field	prob. pop. code
variability	✓	✗	✓
noise correlations	✓	✗	✓
stimulus-dependent variability & correlations	✓	✗	✗

NEURAL REPRESENTATIONS OF UNCERTAINTY
accounting for neural data

	sampling	mean-field	prob. pop. code
variability	✓	✗	✓
noise correlations	✓	✗	✓
stimulus-dependent variability & correlations	✓	✗	✗
spontaneous activity	✓	✗	✗

NEURAL REPRESENTATIONS OF UNCERTAINTY
accounting for neural data

	sampling	mean-field	prob. pop. code
variability	✓	✗	✓
noise correlations	✓	✗	✓
stimulus-dependent variability & correlations	✓	✗	✗
spontaneous activity	✓	✗	✗
correlation of (signal, noise & spont) correlations	✓	✗	–

NEURAL REPRESENTATIONS OF UNCERTAINTY
accounting for neural data

	sampling	mean-field	prob. pop. code
variability	✓	✗	✓
noise correlations	✓	✗	✓
stimulus-dependent variability & correlations	✓	✗	✗
spontaneous activity	✓	✗	✗
correlation of (signal, noise & spont) correlations	✓	✗	–
transients	✓	✓	?

NEURAL REPRESENTATIONS OF UNCERTAINTY
accounting for neural data

	sampling	mean-field	prob. pop. code
variability	✓	✗	✓
noise correlations	✓	✗	✓
stimulus-dependent variability & correlations	✓	✗	✗
spontaneous activity	✓	✗	✗
correlation of (signal, noise & spont) correlations	✓	✗	–
transients	✓	✓	?
oscillations	✓	?	?

NEURAL REPRESENTATIONS OF UNCERTAINTY
accounting for neural data

	sampling	mean-field	prob. pop. code
variability	✓	✗	✓
noise correlations	✓	✗	✓
stimulus-dependent variability & correlations	✓	✗	✗
spontaneous activity	✓	✗	✗
correlation of (signal, noise & spont) correlations	✓	✗	—
transients	✓	✓	?
oscillations	✓	?	?
ramping(?) for evidence integration	?	✓	✓

NEURAL REPRESENTATIONS OF UNCERTAINTY
accounting for neural data

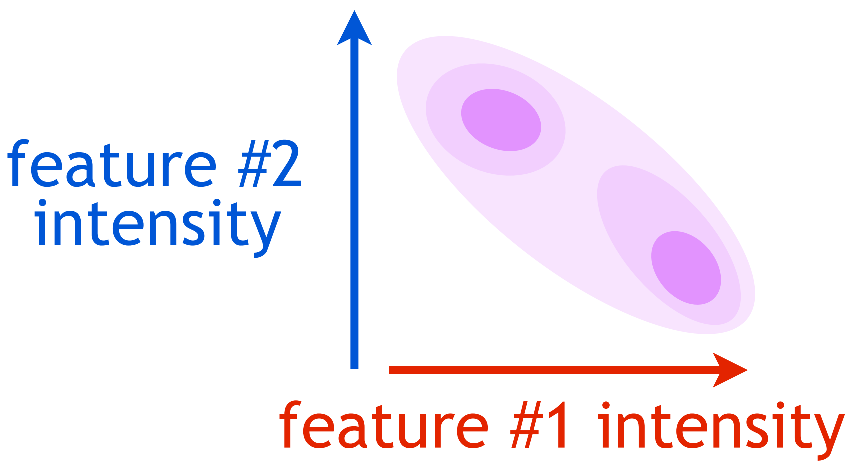
	sampling	mean-field	prob. pop. code
variability	✓	✗	✓
noise correlations	✓	✗	✓
stimulus-dependent variability & correlations	✓	✗	✗
spontaneous activity	✓	✗	✗
correlation of (signal, noise & spont) correlations	✓	✗	–
transients	✓	✓	?
oscillations	✓	?	?
ramping(?) for evidence integration	?	✓	✓
spiking neurons	✓	✓	✓

A PSYCHOPHYSICAL HALLMARK OF SAMPLING

Lengyel et al, arXiv 2015

A PSYCHOPHYSICAL HALLMARK OF SAMPLING

Lengyel et al, arXiv 2015



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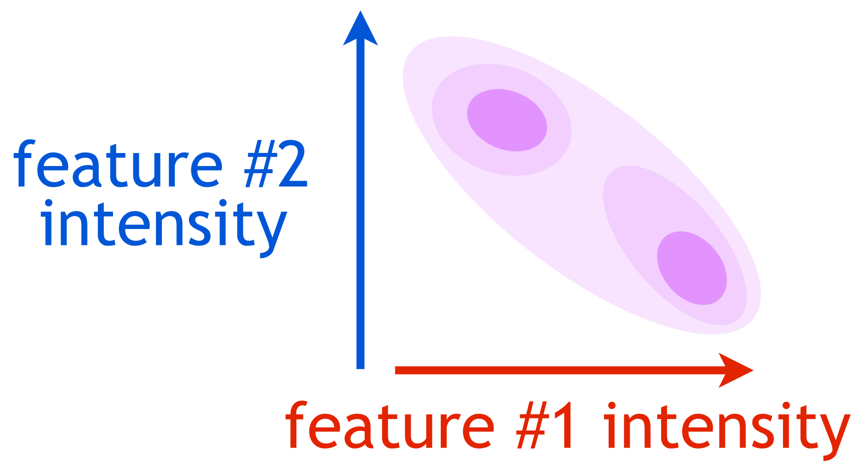
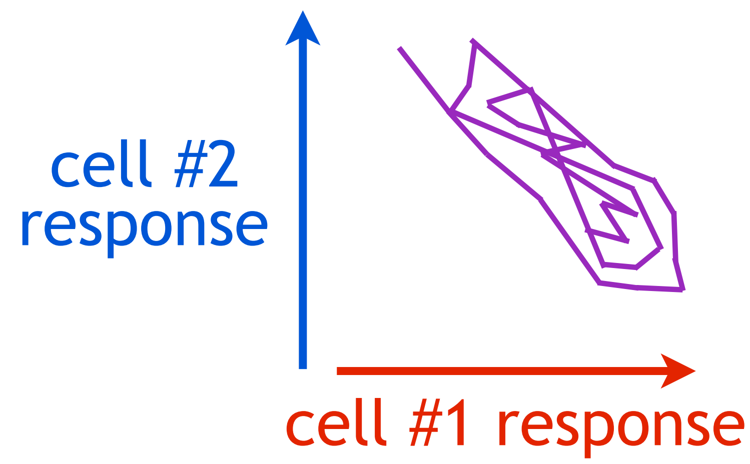
Lengyel et al, arXiv 2015





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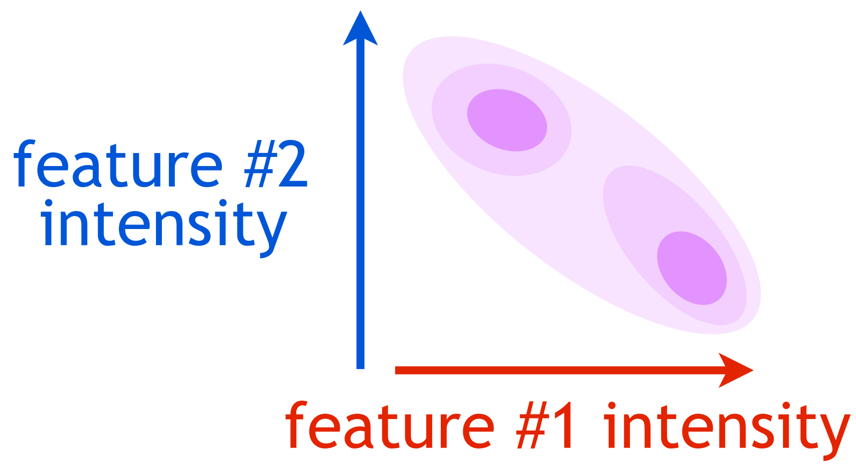

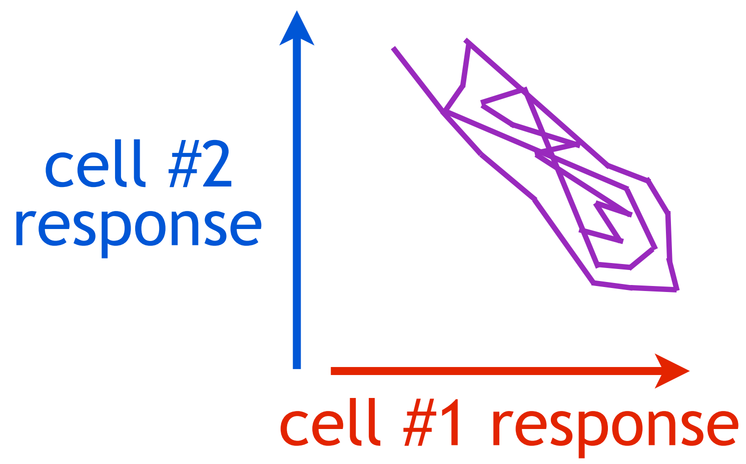

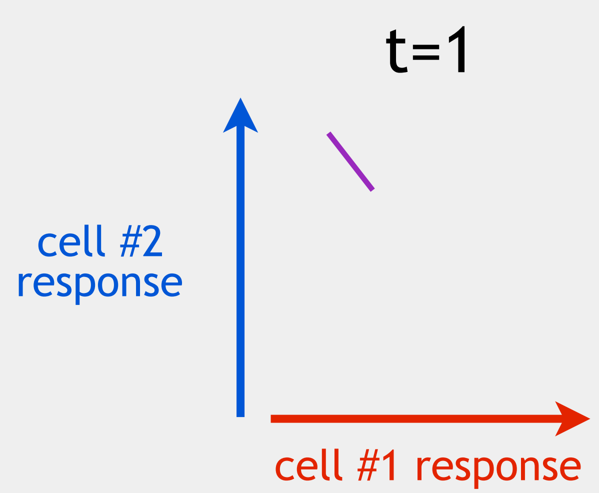

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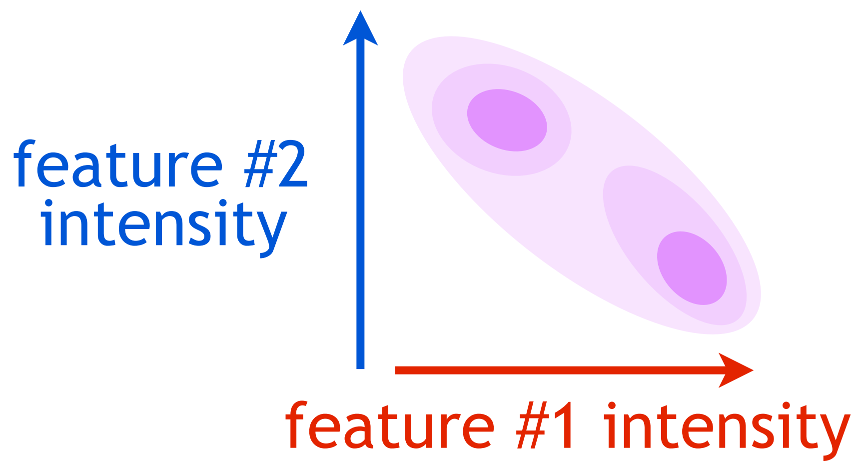

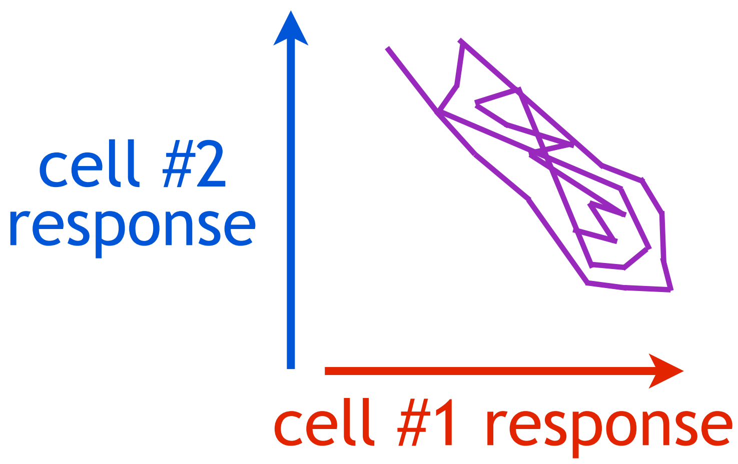

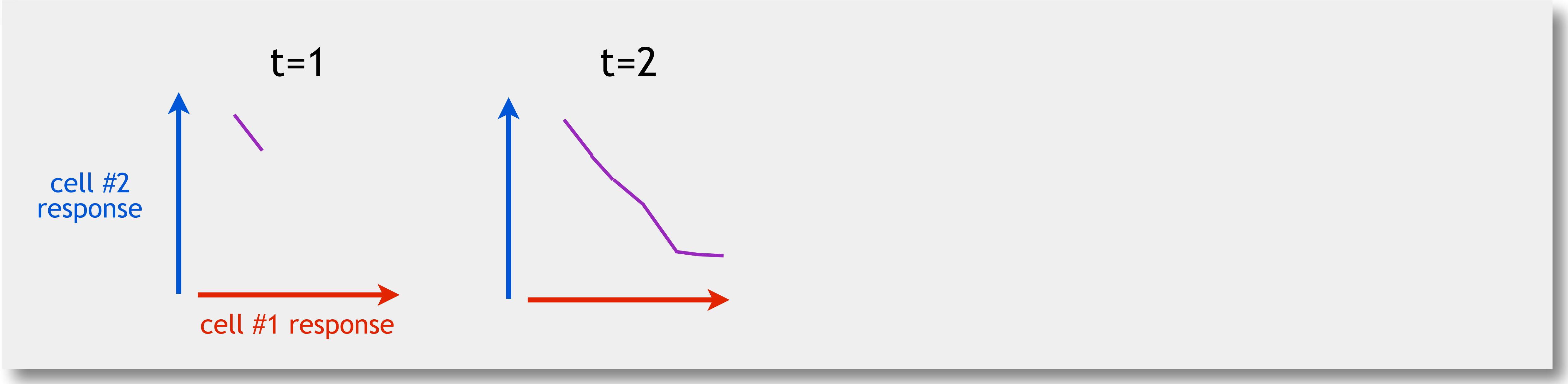

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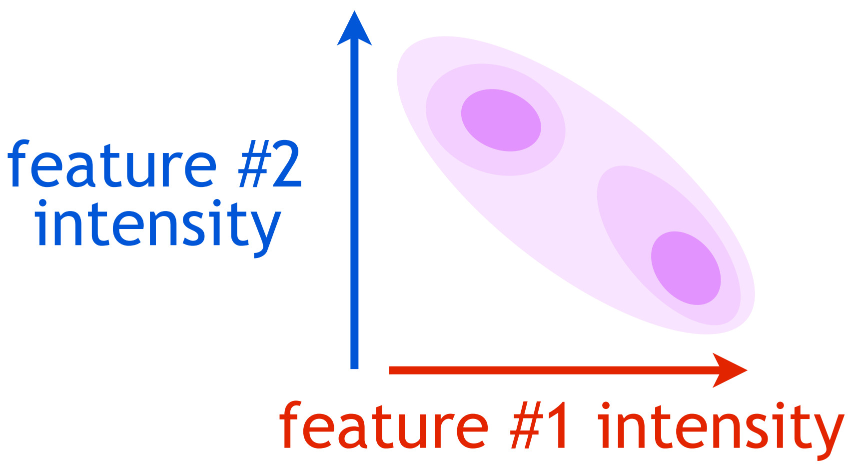

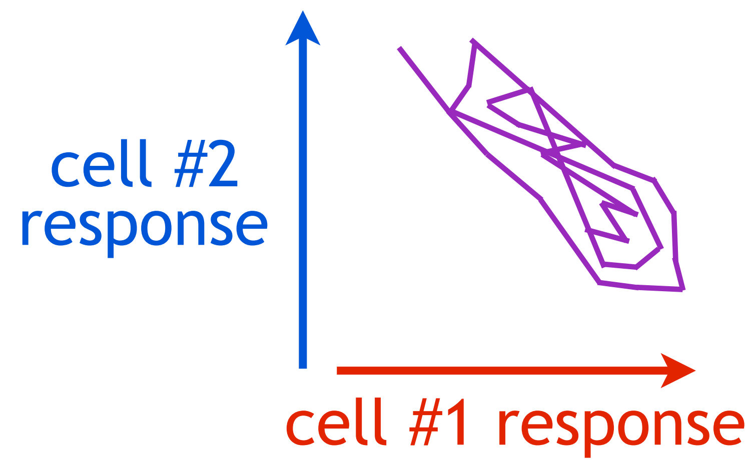

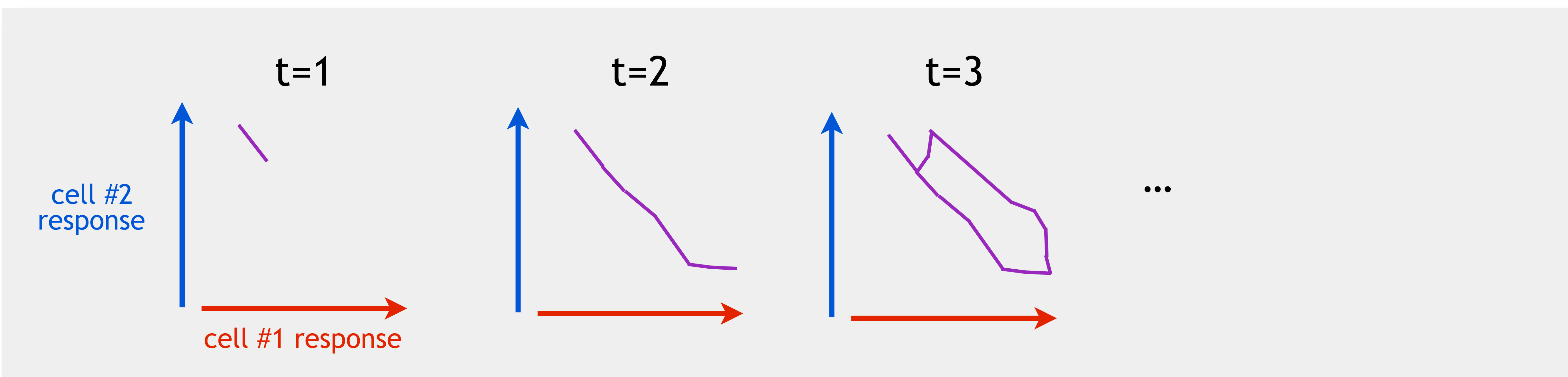

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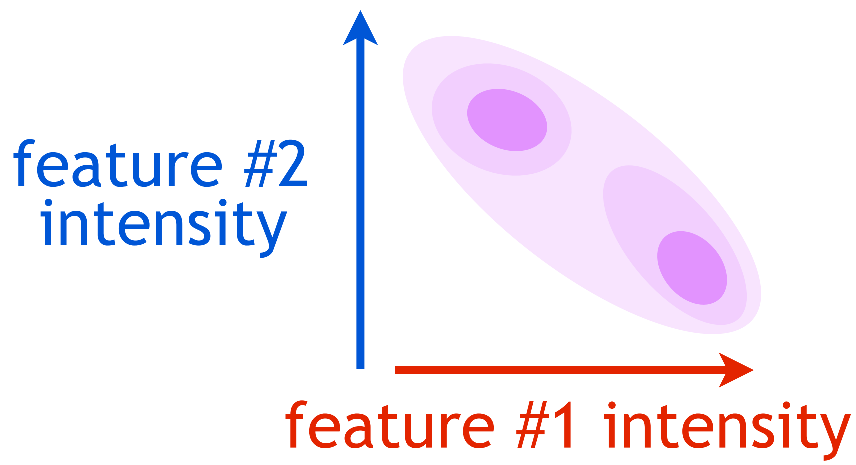
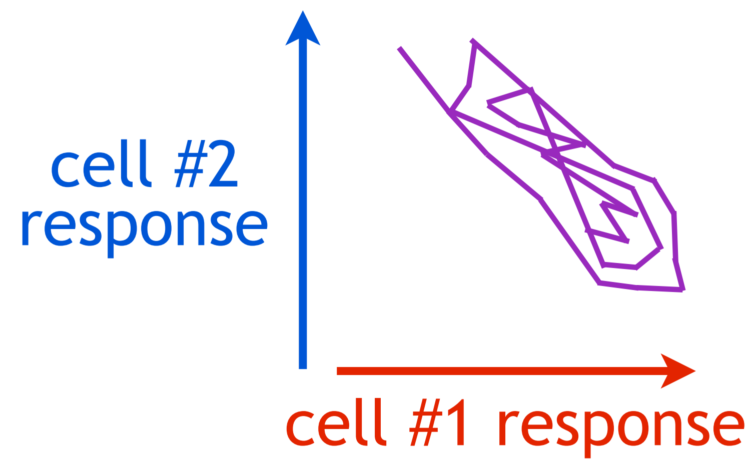
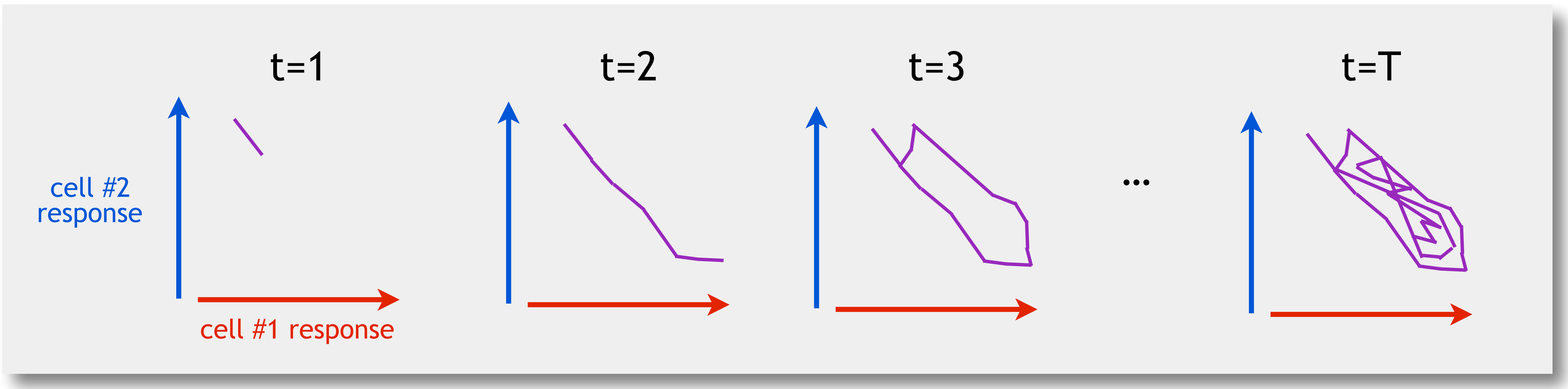
Lengyel et al, arXiv 2015







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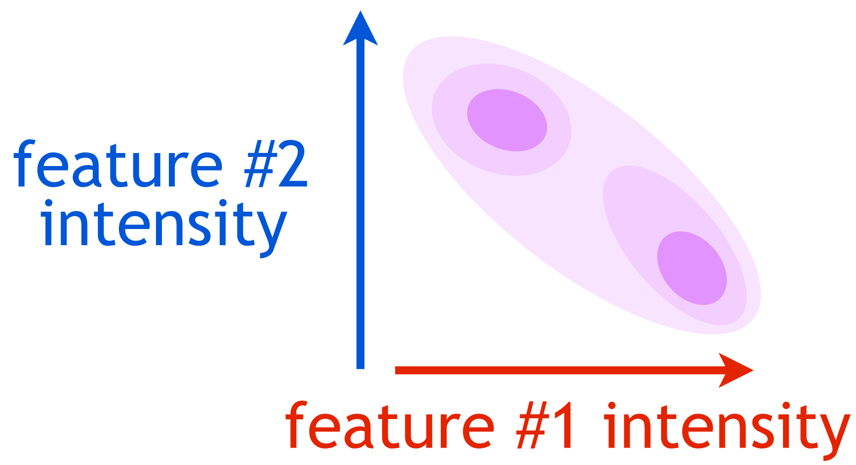
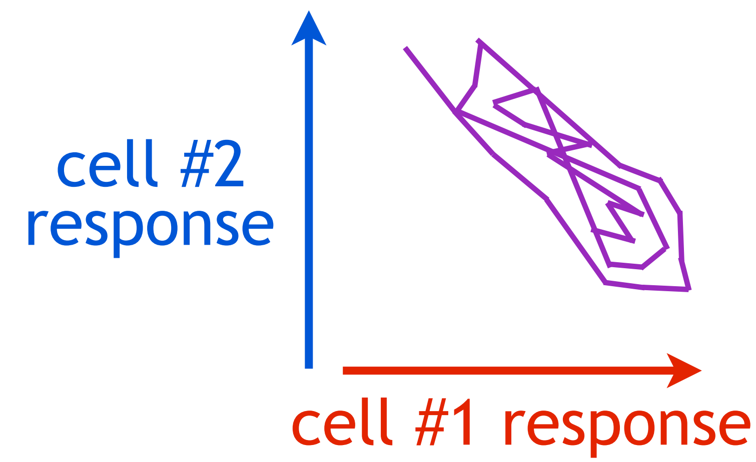
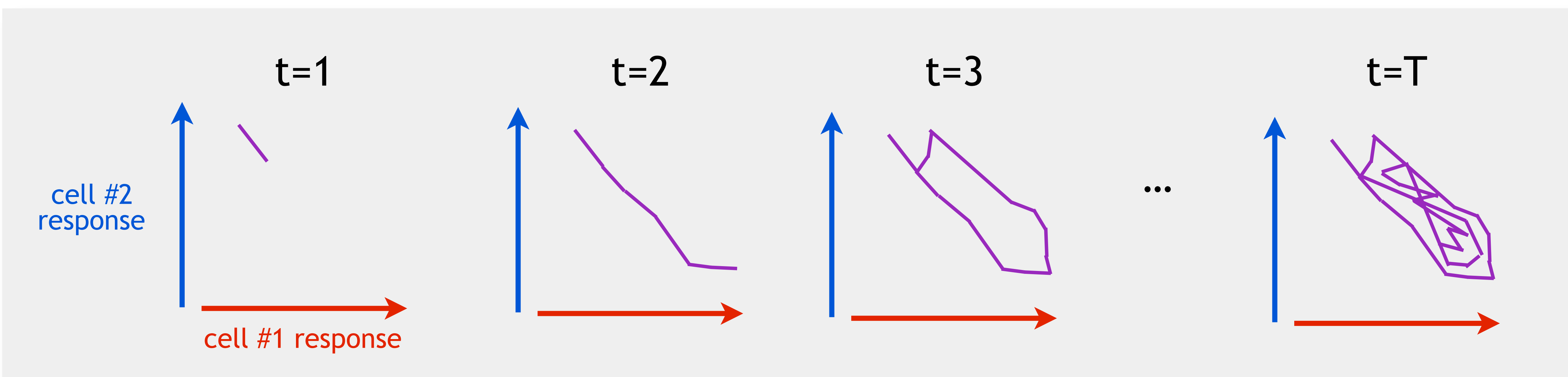
Lengyel et al, arXiv 2015







A GRADUAL REFINEMENT OF THE REPRESENTATION OF UNCERTAINTY

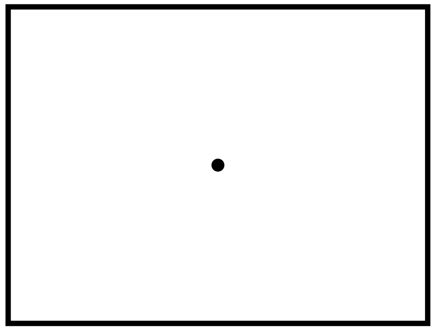
A PSYCHOPHYSICAL TEST

Lengyel et al, arXiv 2015

A PSYCHOPHYSICAL TEST

Lengyel et al, arXiv 2015

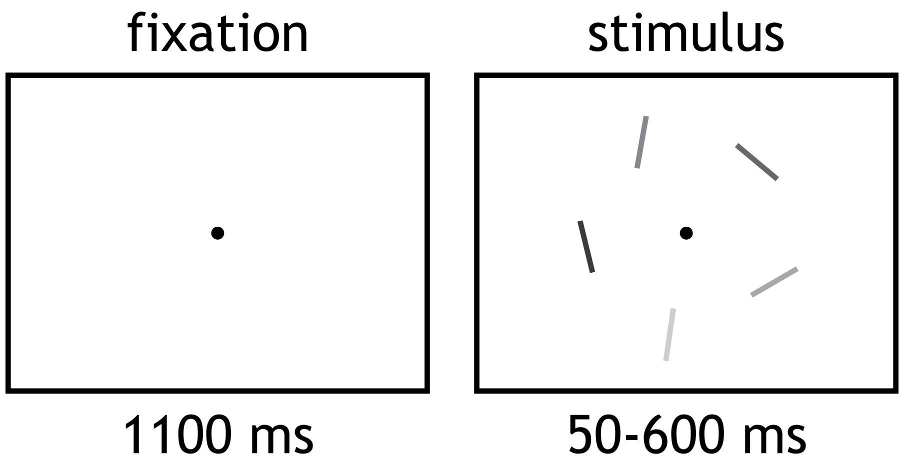
fixation



1100 ms

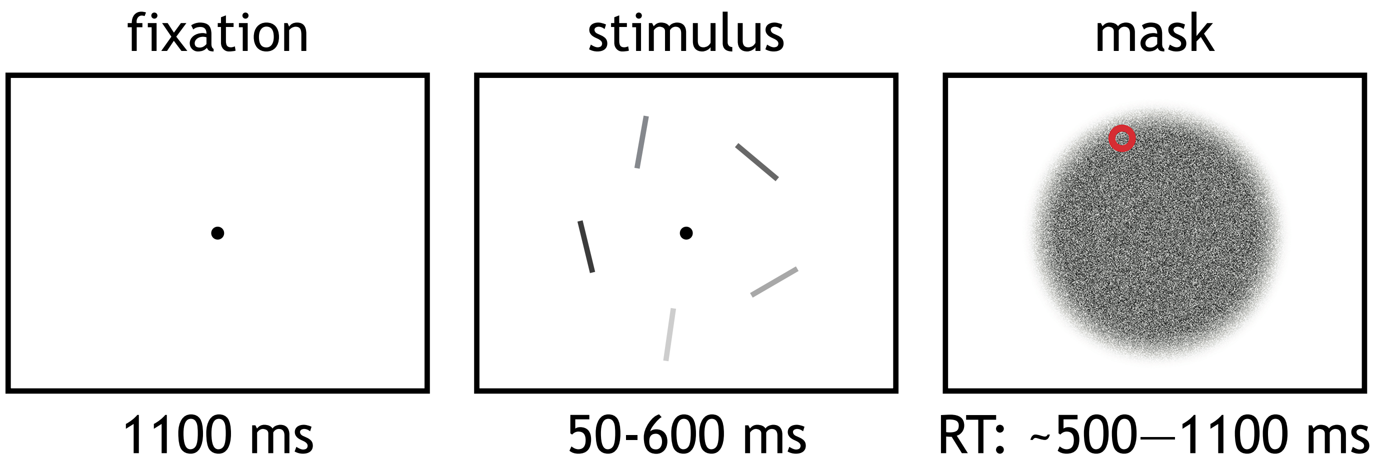
A PSYCHOPHYSICAL TEST

Lengyel et al, arXiv 2015



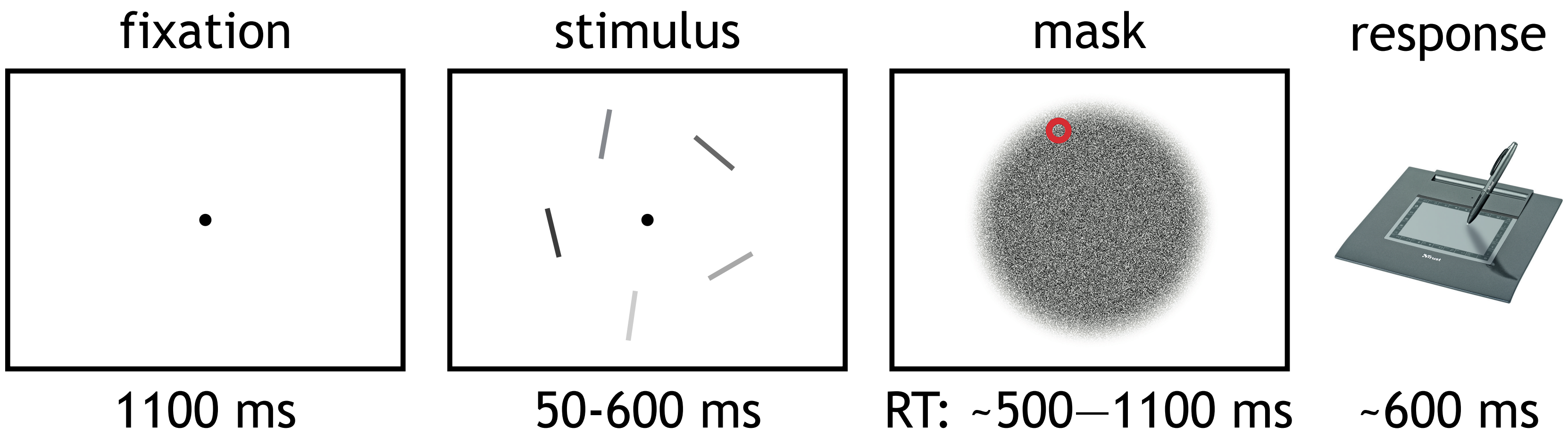
A PSYCHOPHYSICAL TEST

Lengyel et al, arXiv 2015



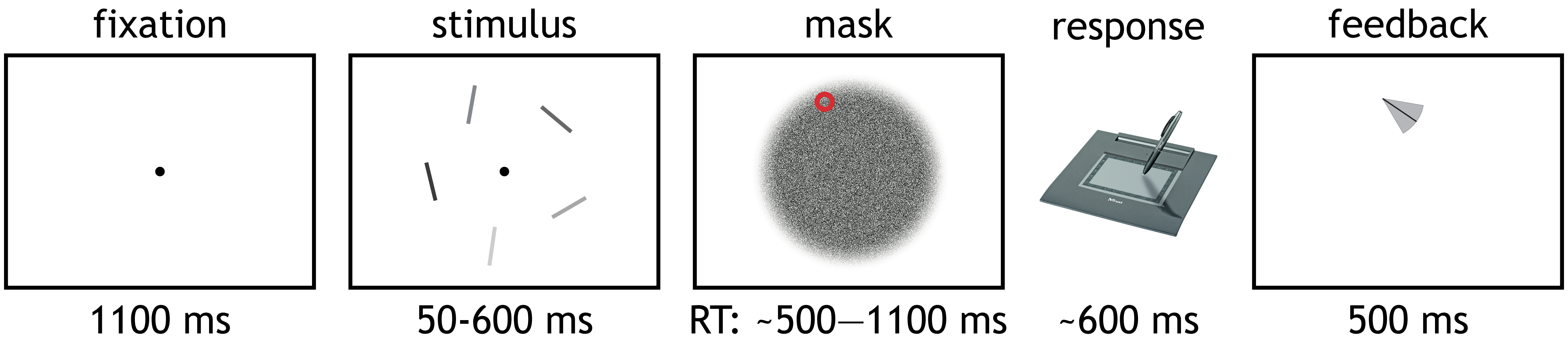
A PSYCHOPHYSICAL TEST

Lengyel et al, arXiv 2015



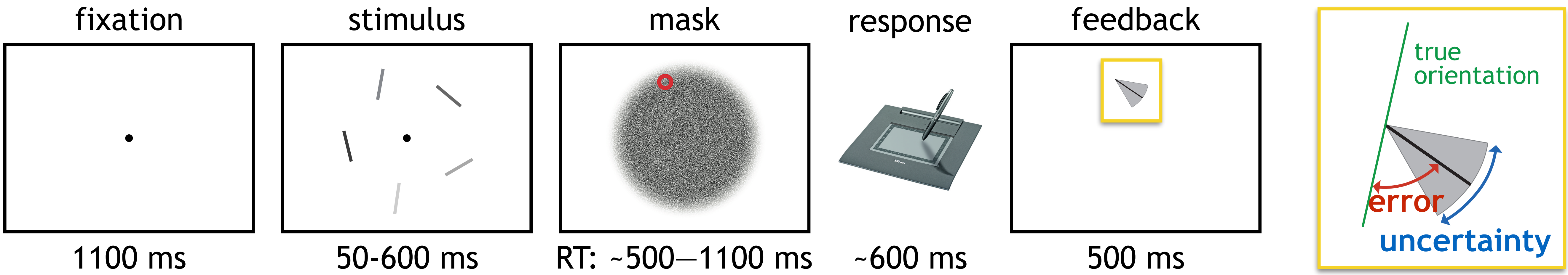
A PSYCHOPHYSICAL TEST

Lengyel et al, arXiv 2015



A PSYCHOPHYSICAL TEST

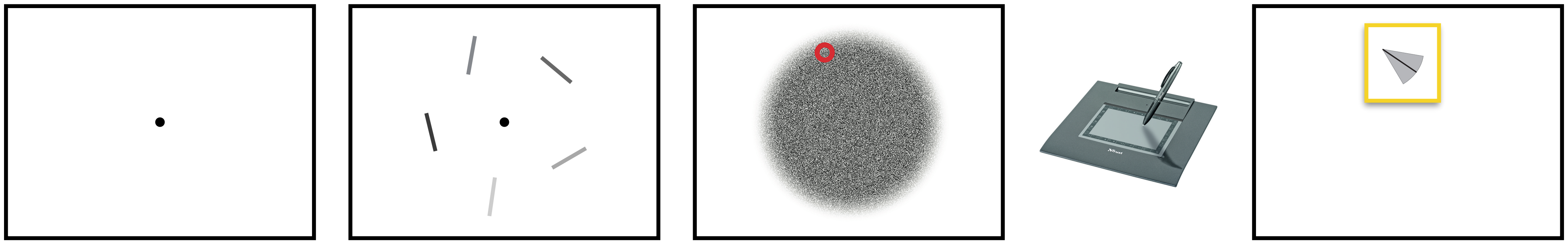
Lengyel et al, arXiv 2015



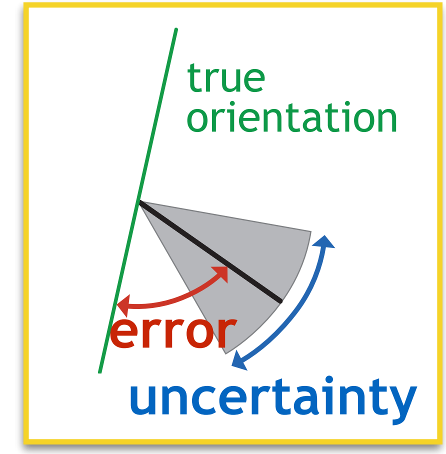
A PSYCHOPHYSICAL TEST

Lengyel et al, arXiv 2015

fixation | stimulus | mask | response | feedback



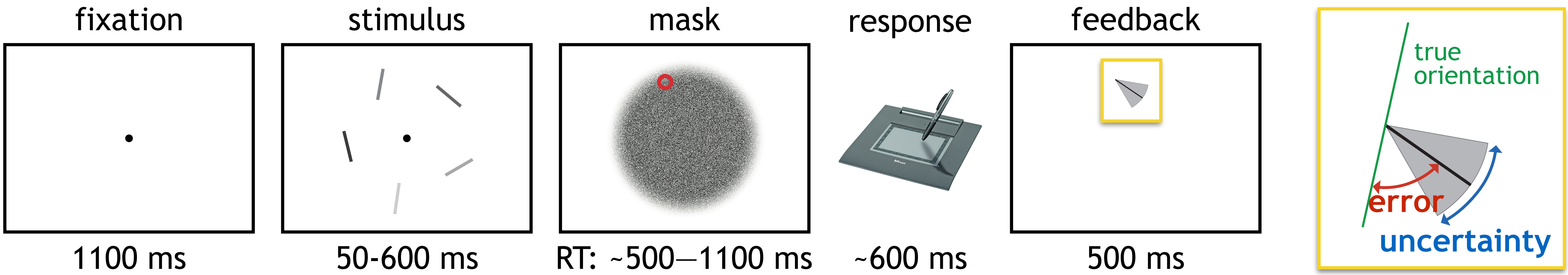
1100 ms | 50-600 ms | RT: ~500—1100 ms | ~600 ms | 500 ms



quality of information available: error (uncertainty)

A PSYCHOPHYSICAL TEST

Lengyel et al, arXiv 2015



quality of information available: error (uncertainty)

quality of probabilistic representation: error-uncertainty *correlation*

A PSYCHOPHYSICAL TEST

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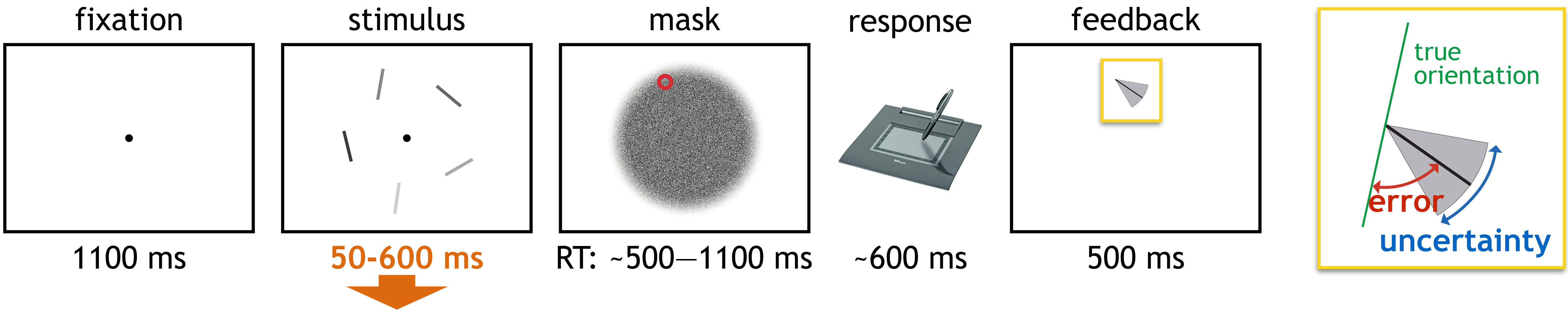

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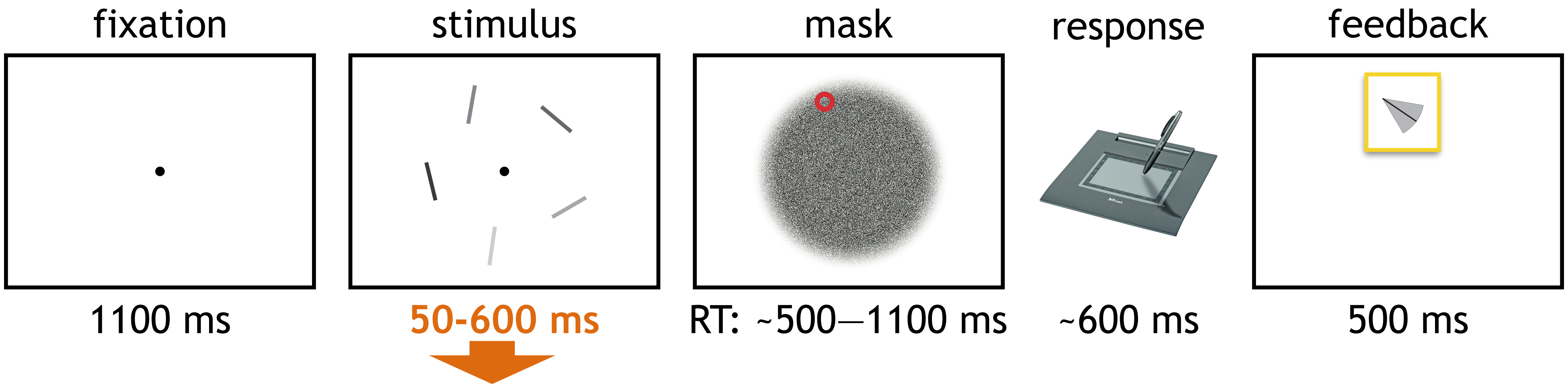

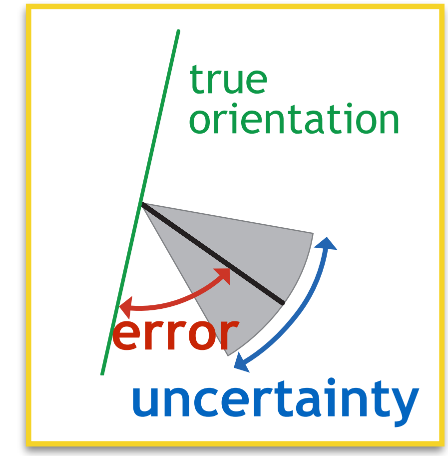

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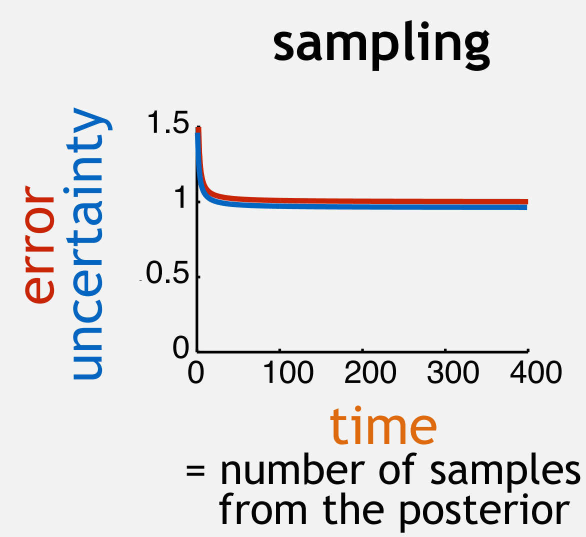
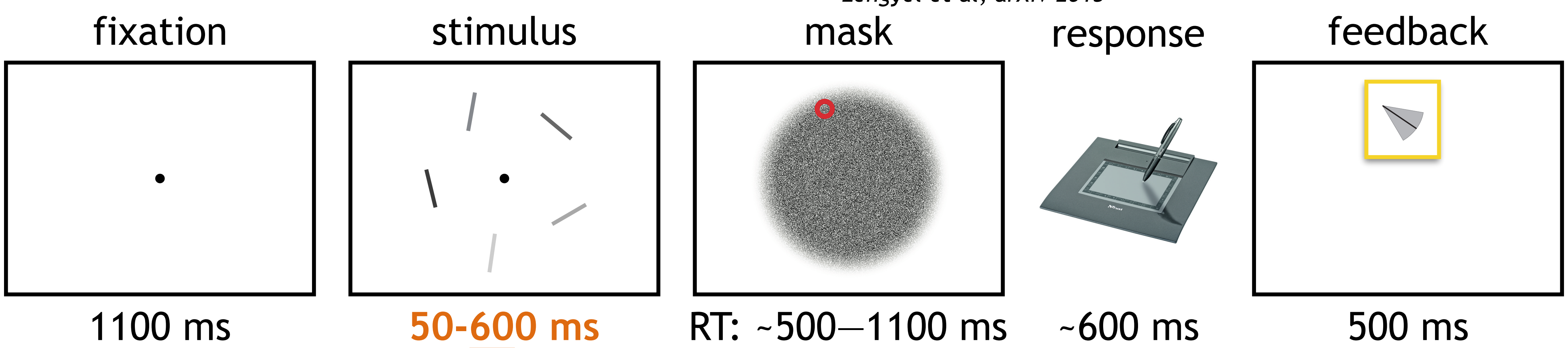
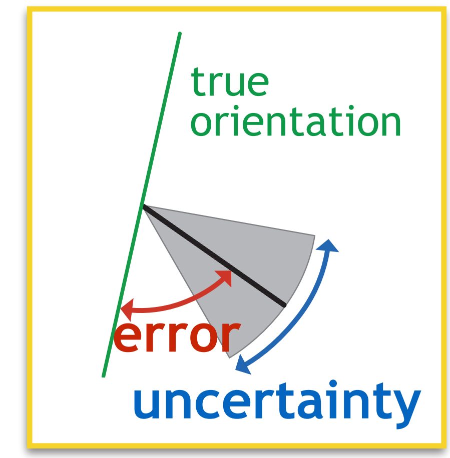


A PSYCHOPHYSICAL TEST

Lengyel et al, arXiv 2015





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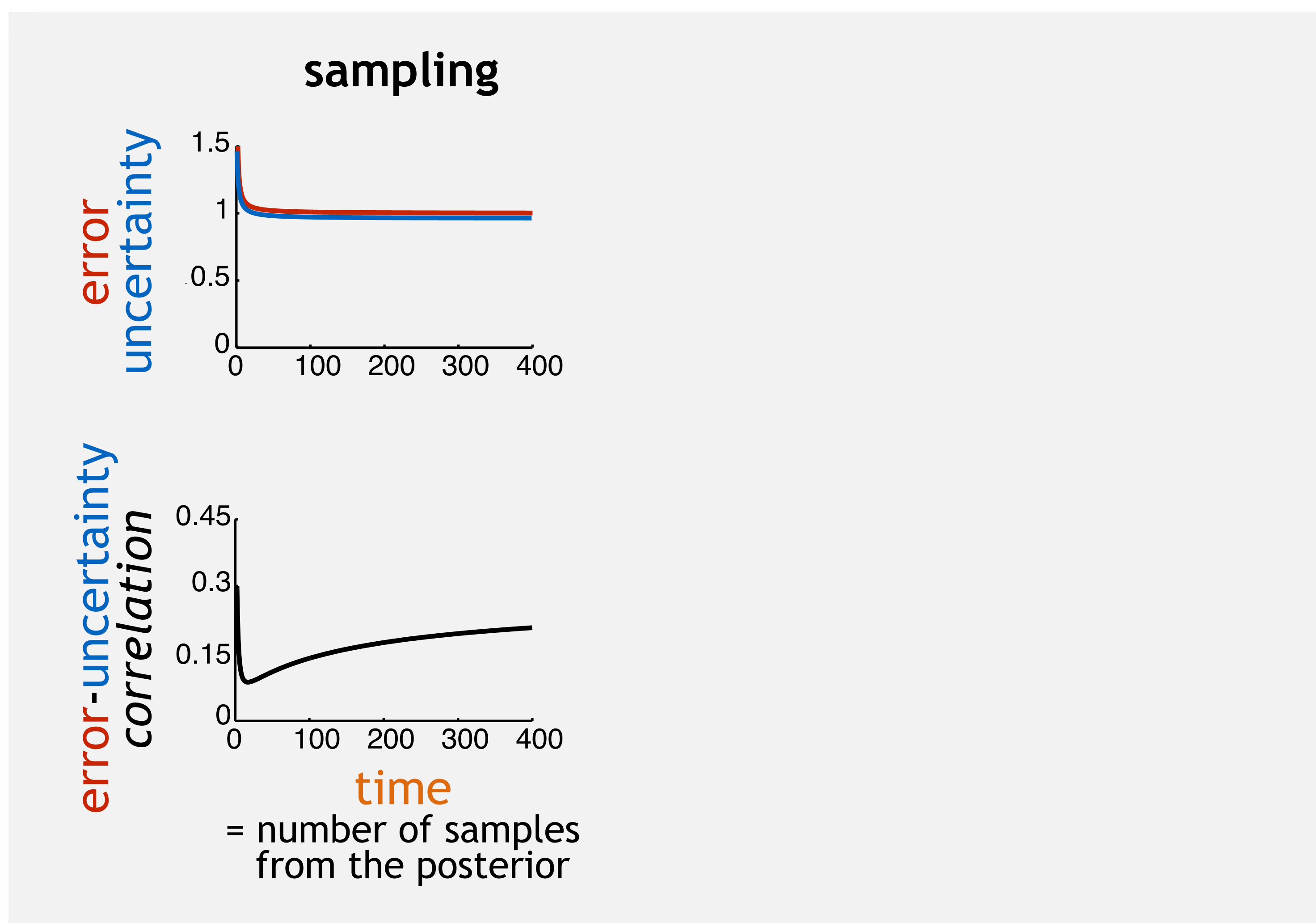

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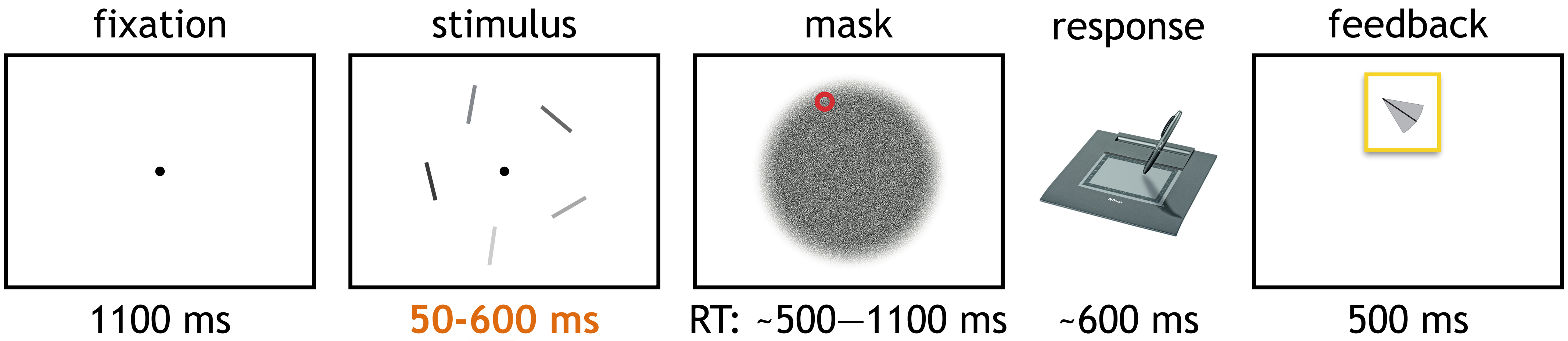
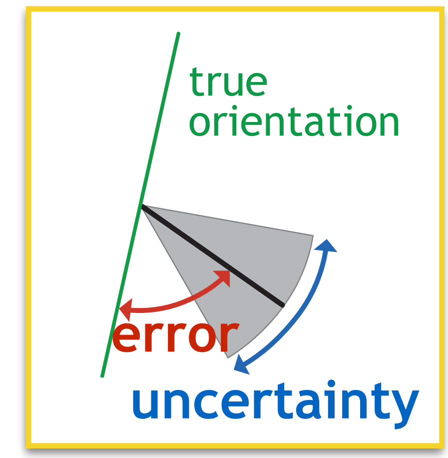
Lengyel et al, arXiv 2015





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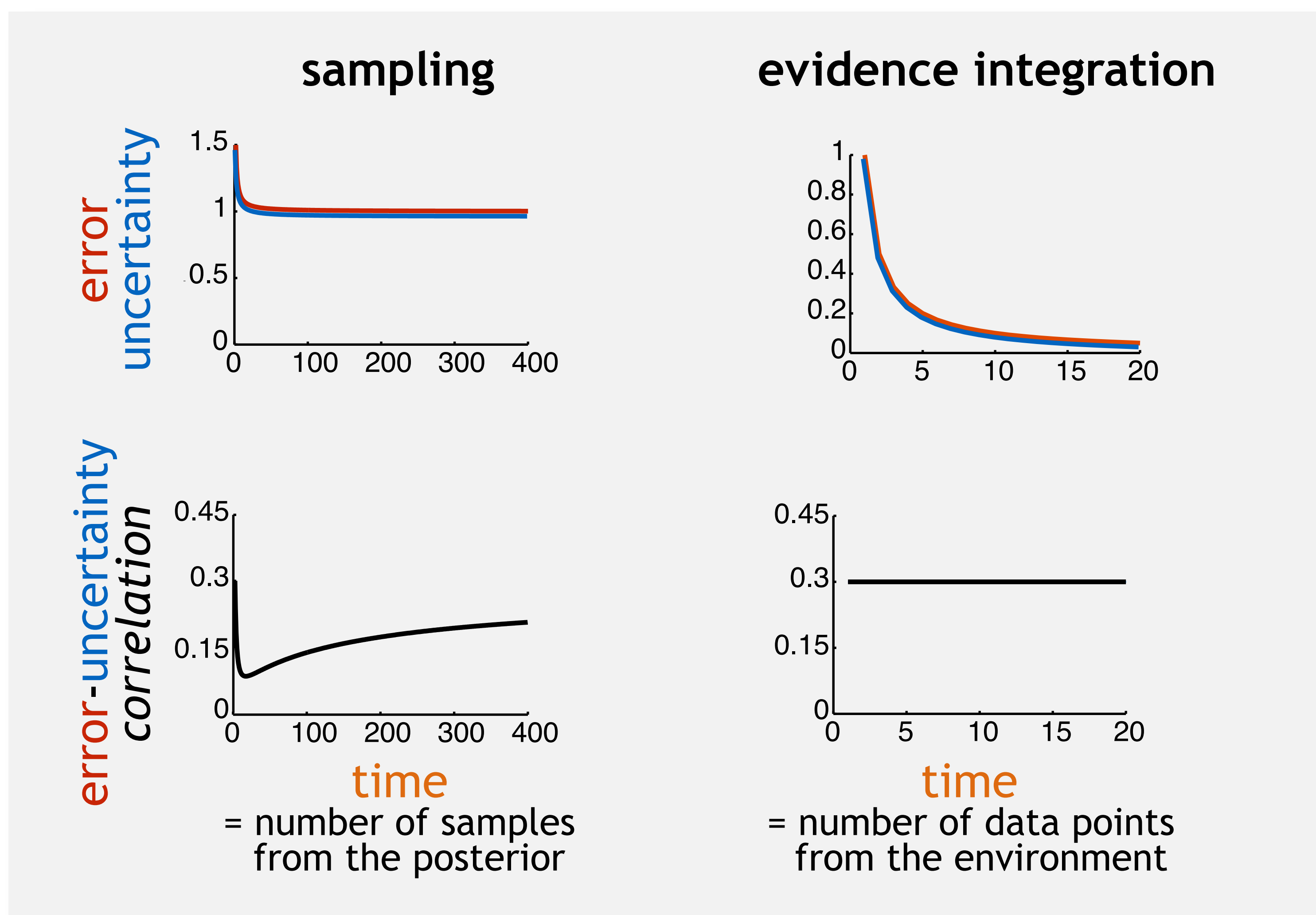
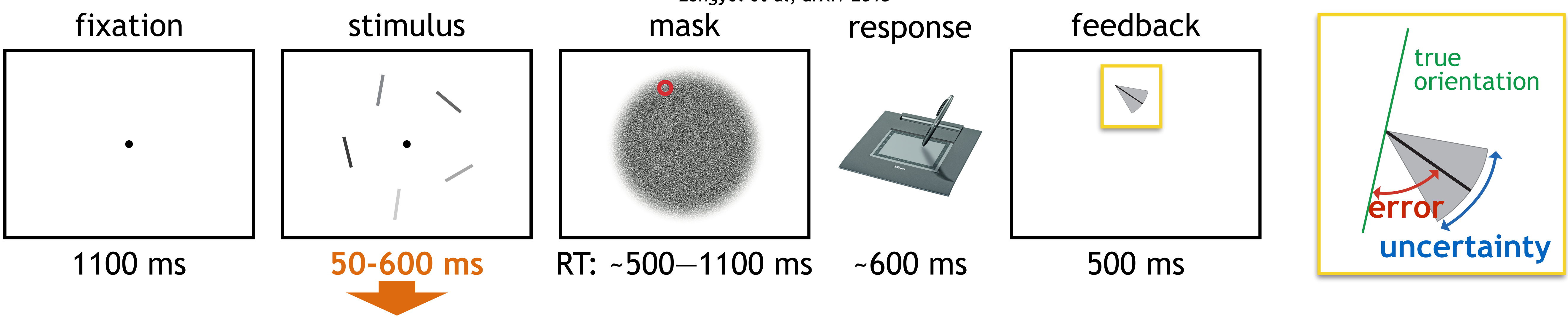


A PSYCHOPHYSICAL TEST

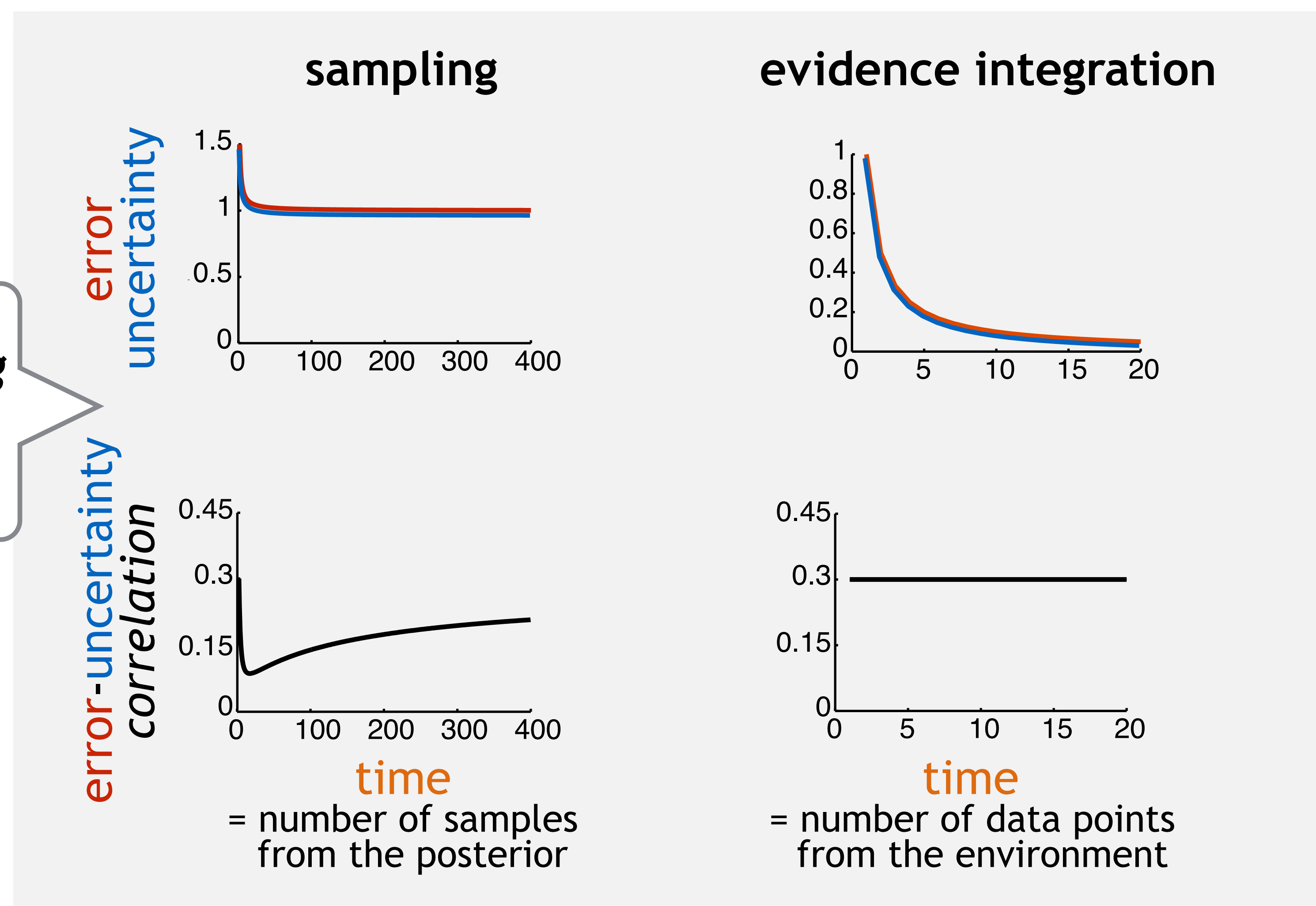
Lengyel et al, arXiv 2015



quality of information available: error (uncertainty)

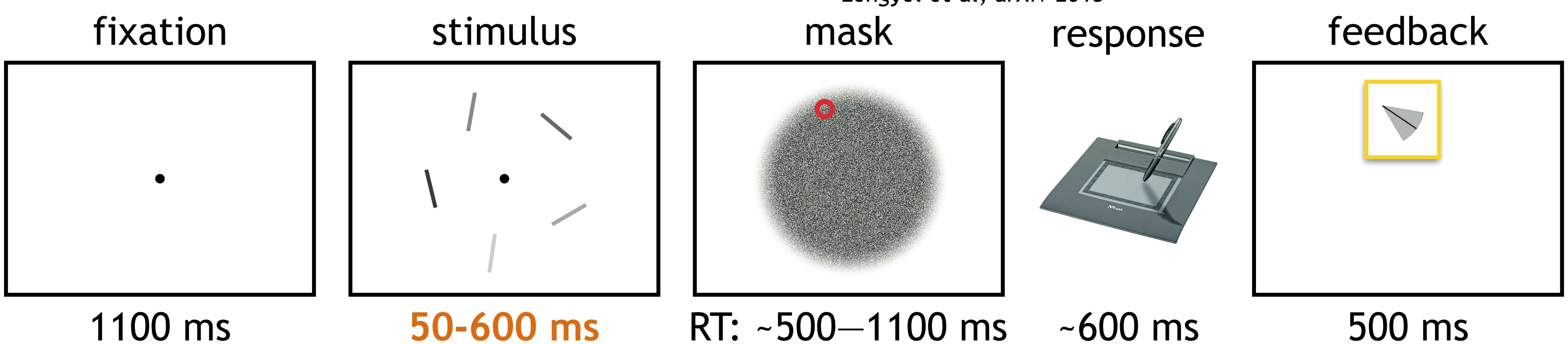
quality of probabilistic representation: error-uncertainty *correlation*

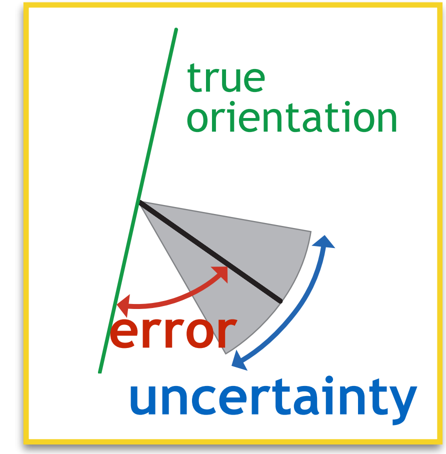
a ***gradually improving*** representation of a ***static*** posterior



A PSYCHOPHYSICAL TEST

Lengyel et al, arXiv 2015





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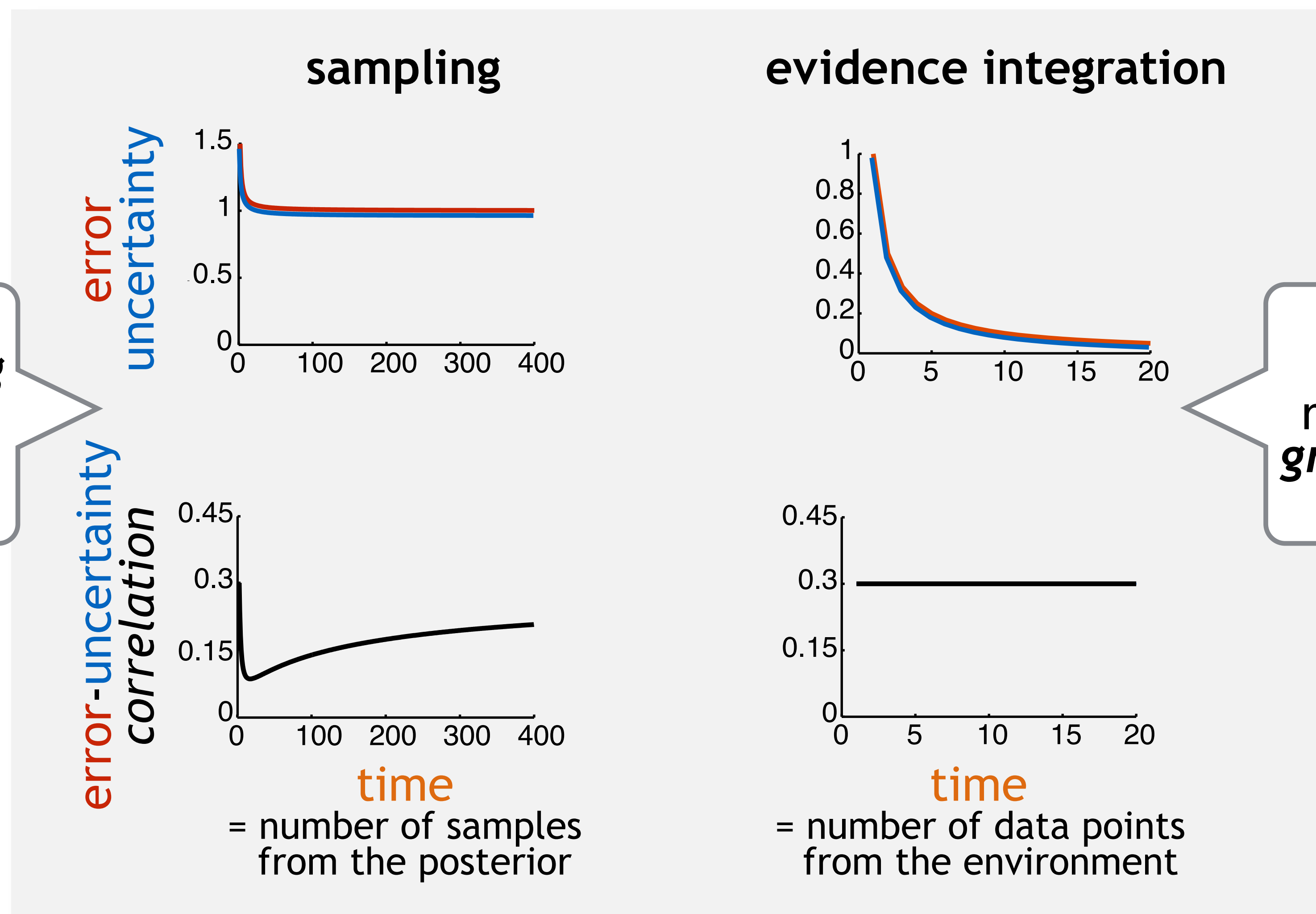
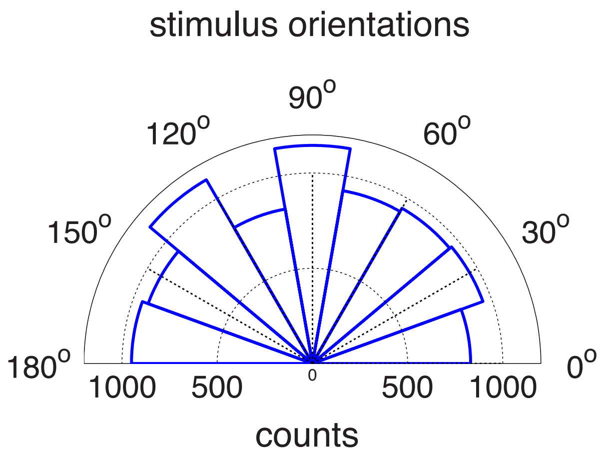


a ***perfect*** representation of a ***gradually improving*** posterior

SANITY CHECKS: BASIC RESPONSE

Lengyel et al, arXiv 2015

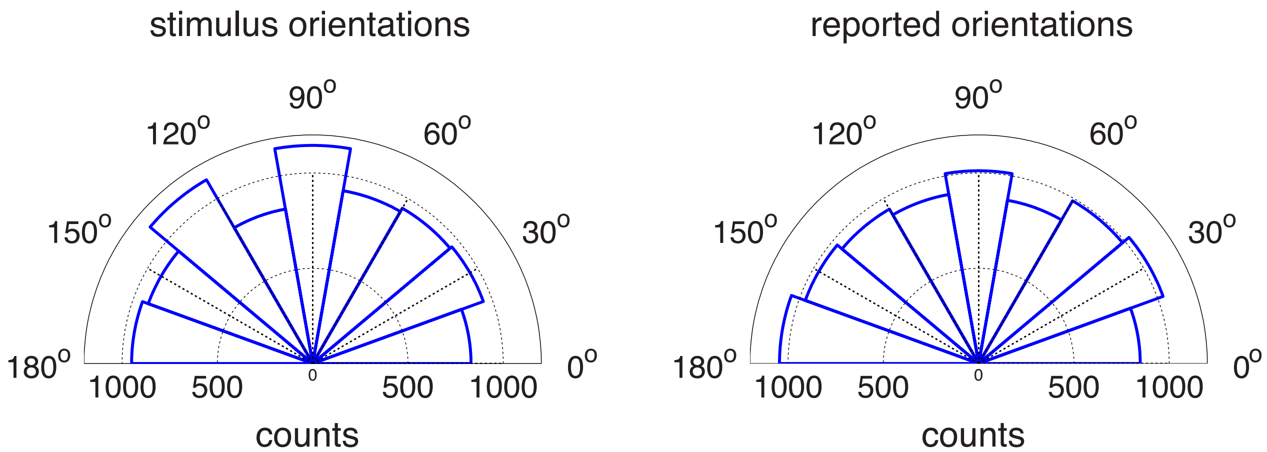
uniform spread



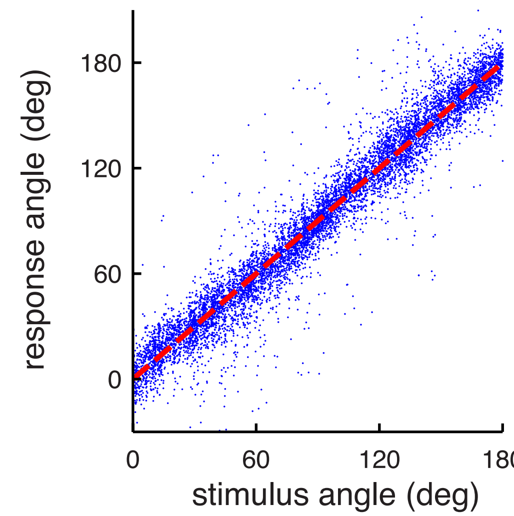
SANITY CHECKS: BASIC RESPONSE

Lengyel et al, arXiv 2015

uniform spread



reliable, no strong bias



SANITY CHECKS: BASIC RESPONSE

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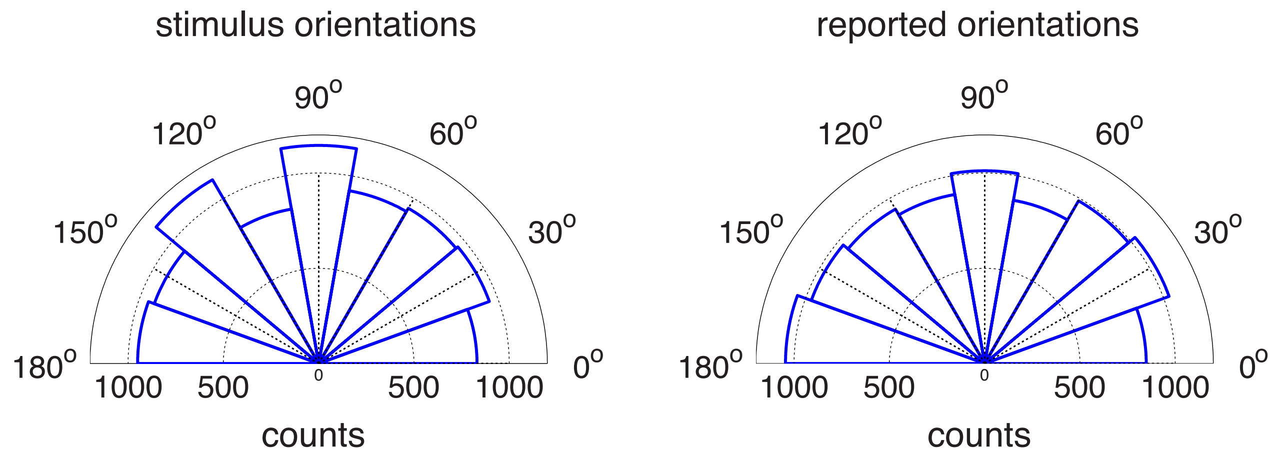

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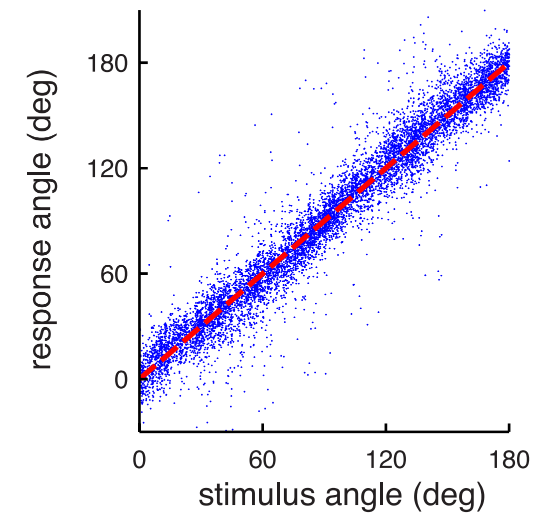
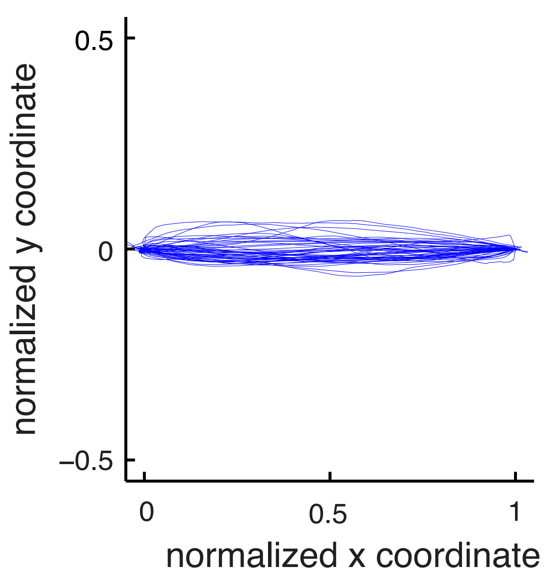


no time to deliberate…



drawing time

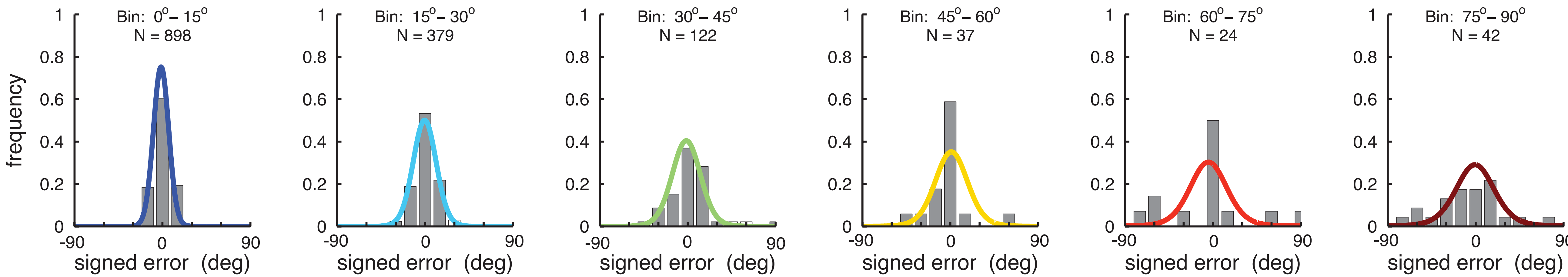
mean: 450±110 ms

std: 160±30 ms

A WELL-CALIBRATED PROBABILISTIC

Lengyel et al, arXiv 2015

a single subject:



A WELL-CALIBRATED PROBABILISTIC

Lengyel et al, arXiv 2015

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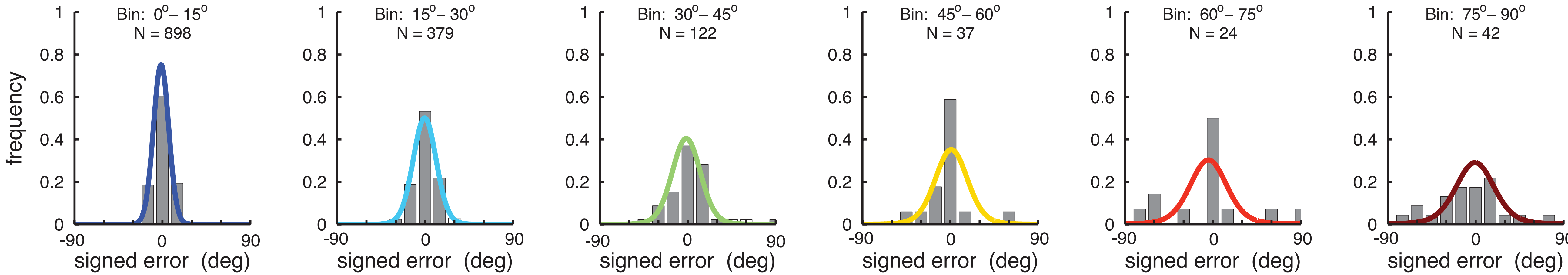
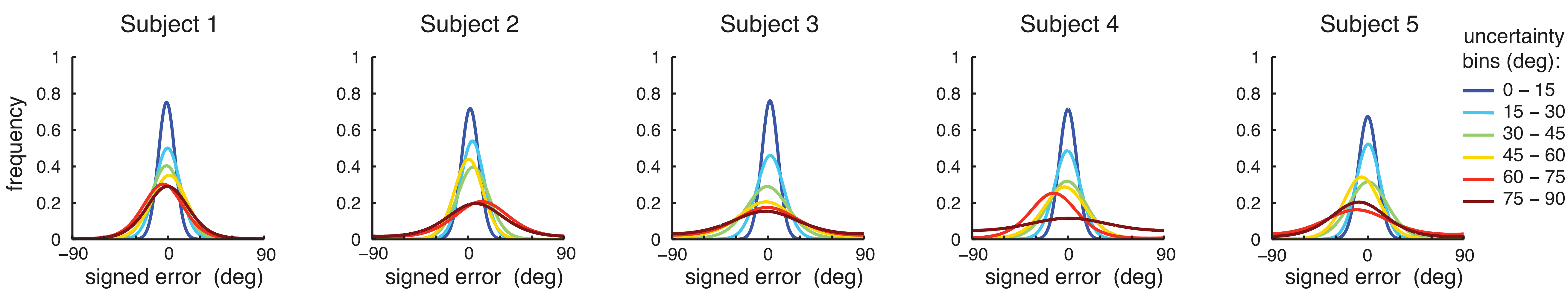


all subjects:



A WELL-CALIBRATED PROBABILISTIC

Lengyel et al, arXiv 2015

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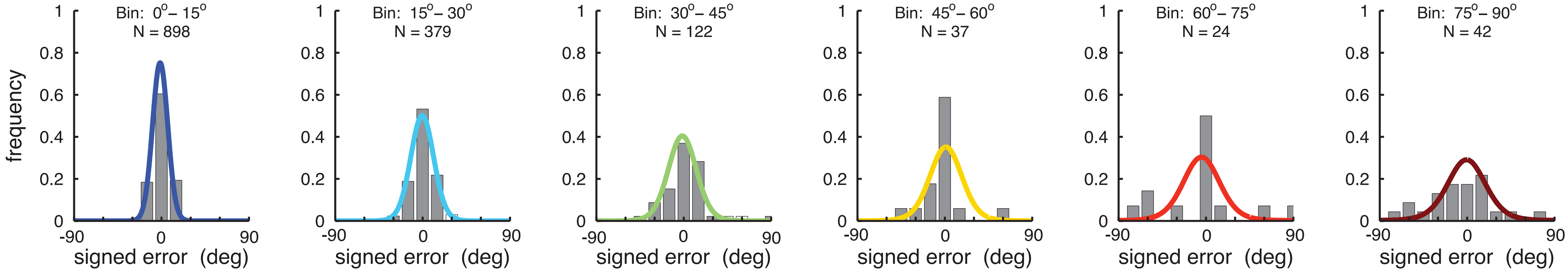

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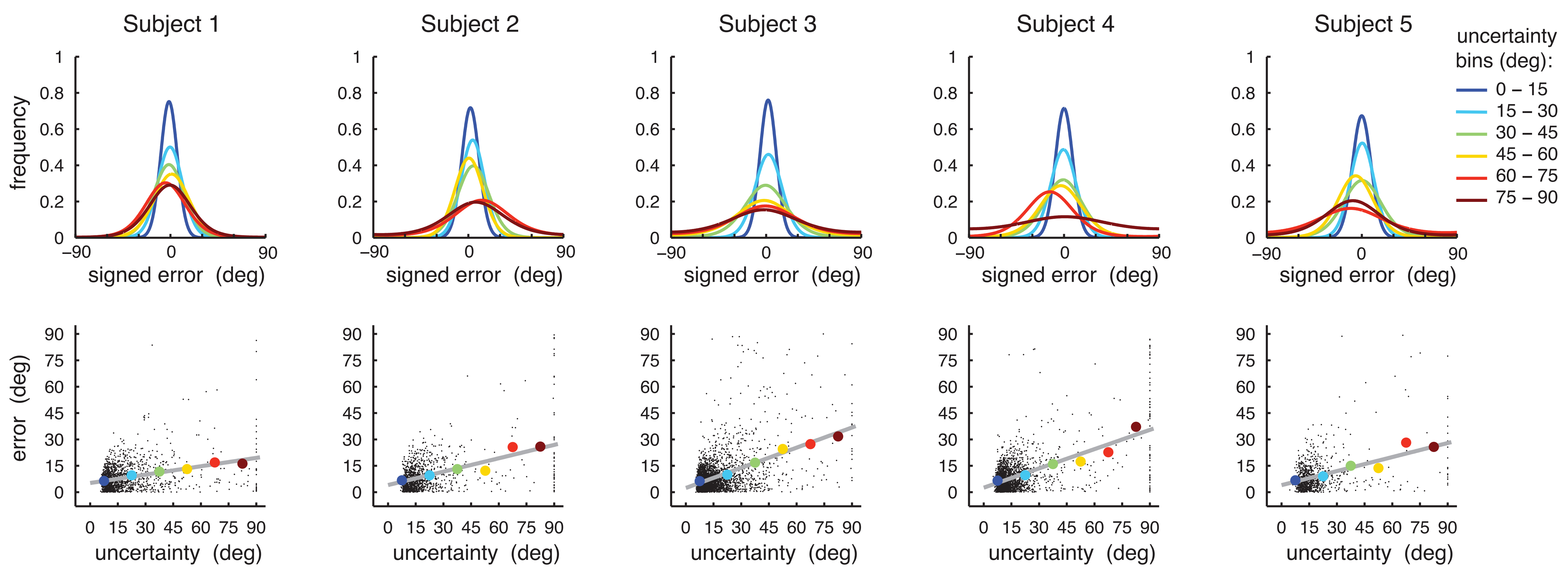

EFFECTS OF TASK DIFFICULTY

Lengyel et al, arXiv 2015

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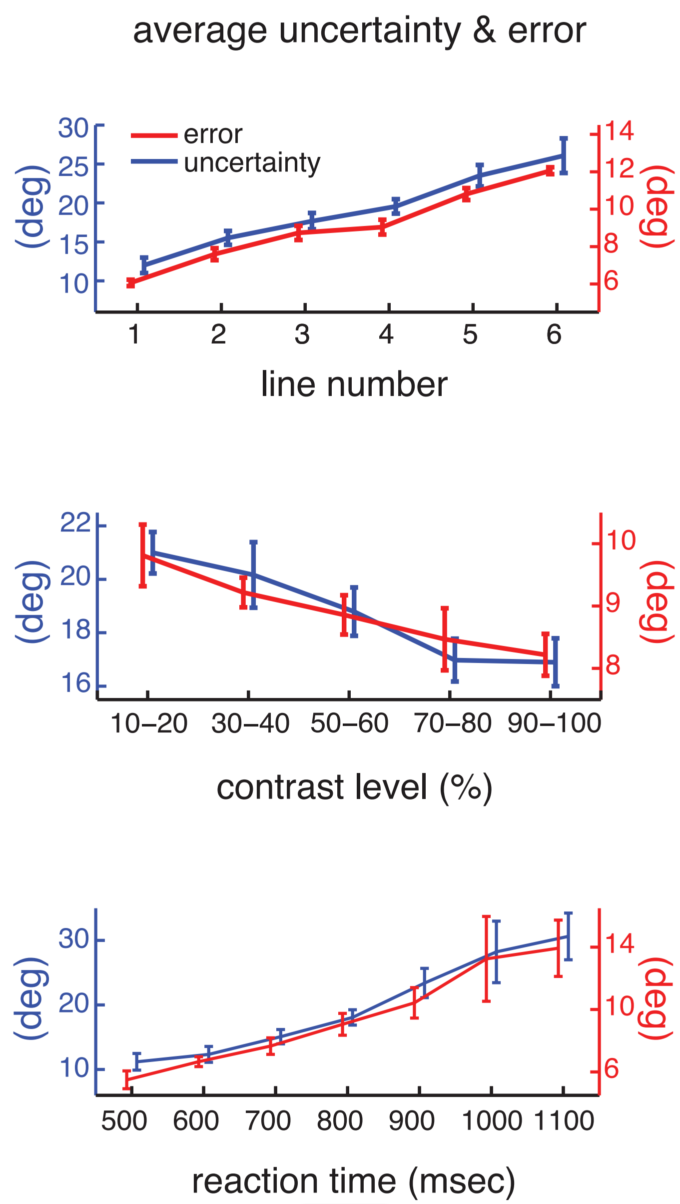
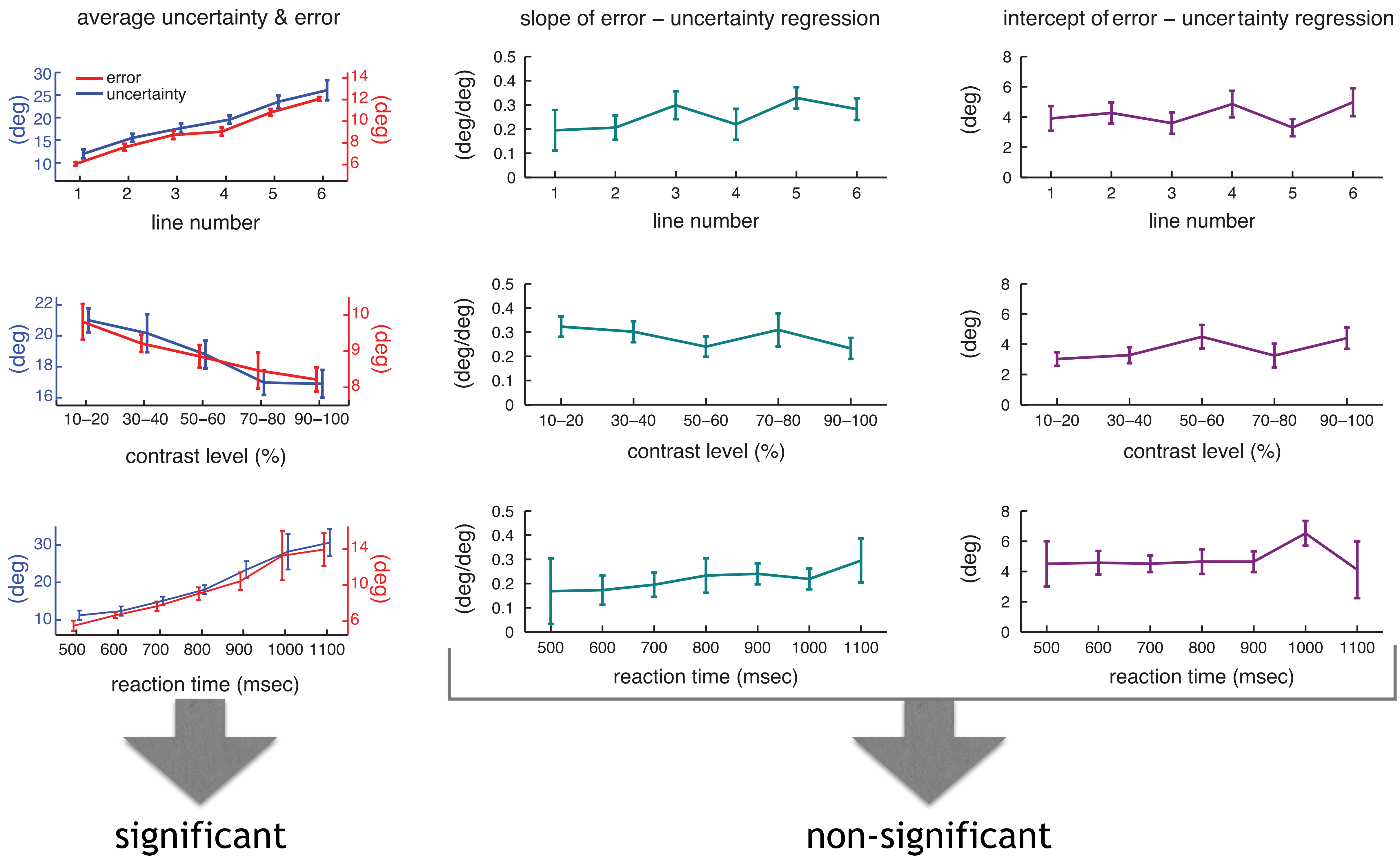
Lengyel et al, arXiv 2015



significant

EFFECTS OF TASK DIFFICULTY

Lengyel et al, arXiv 2015



EFFECTS OF TASK DIFFICULTY

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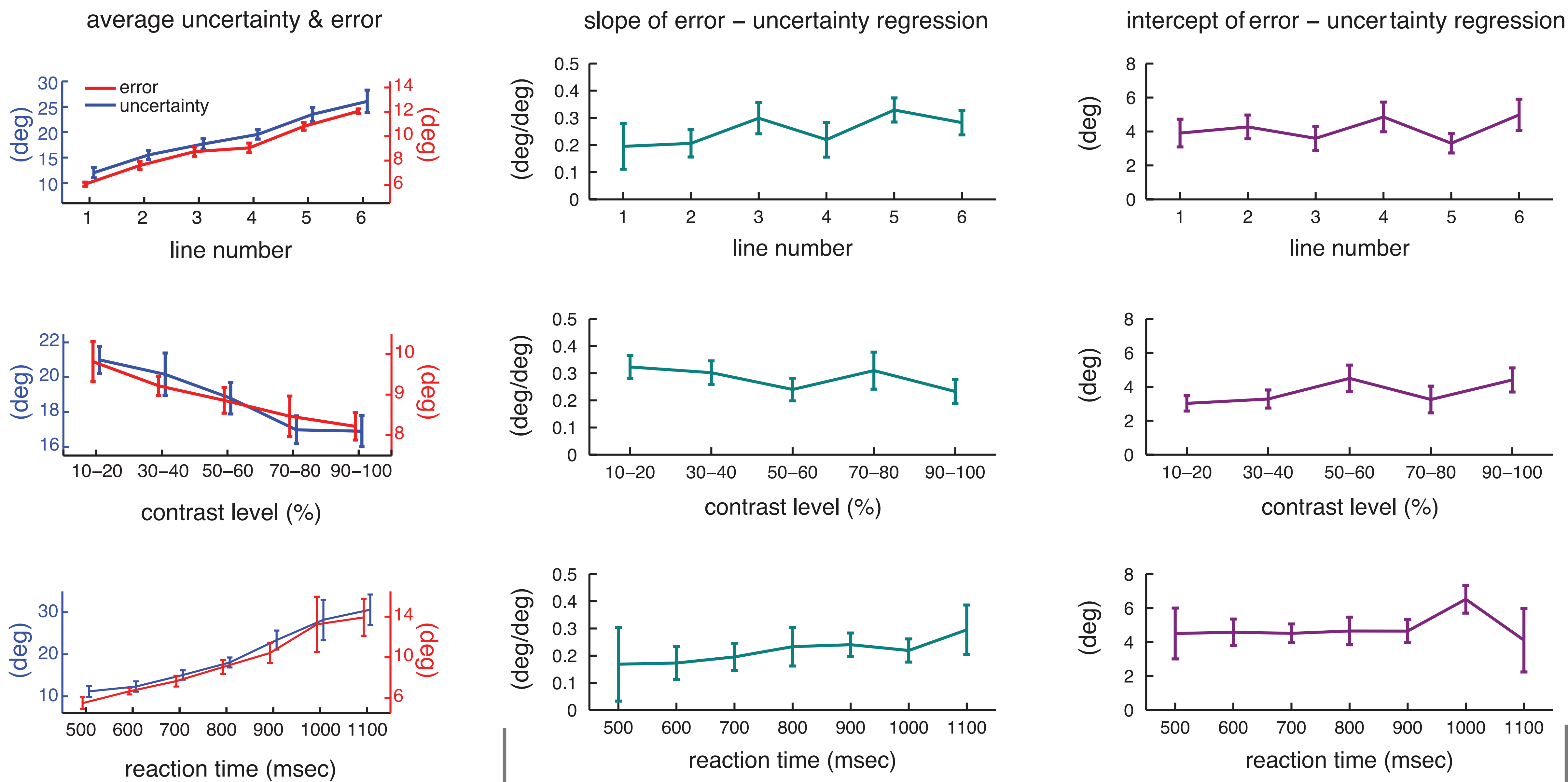
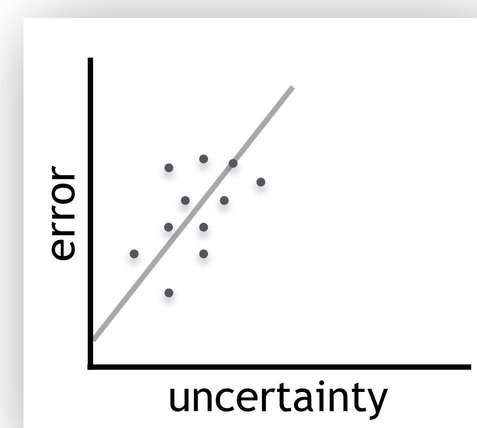


significant

non-significant



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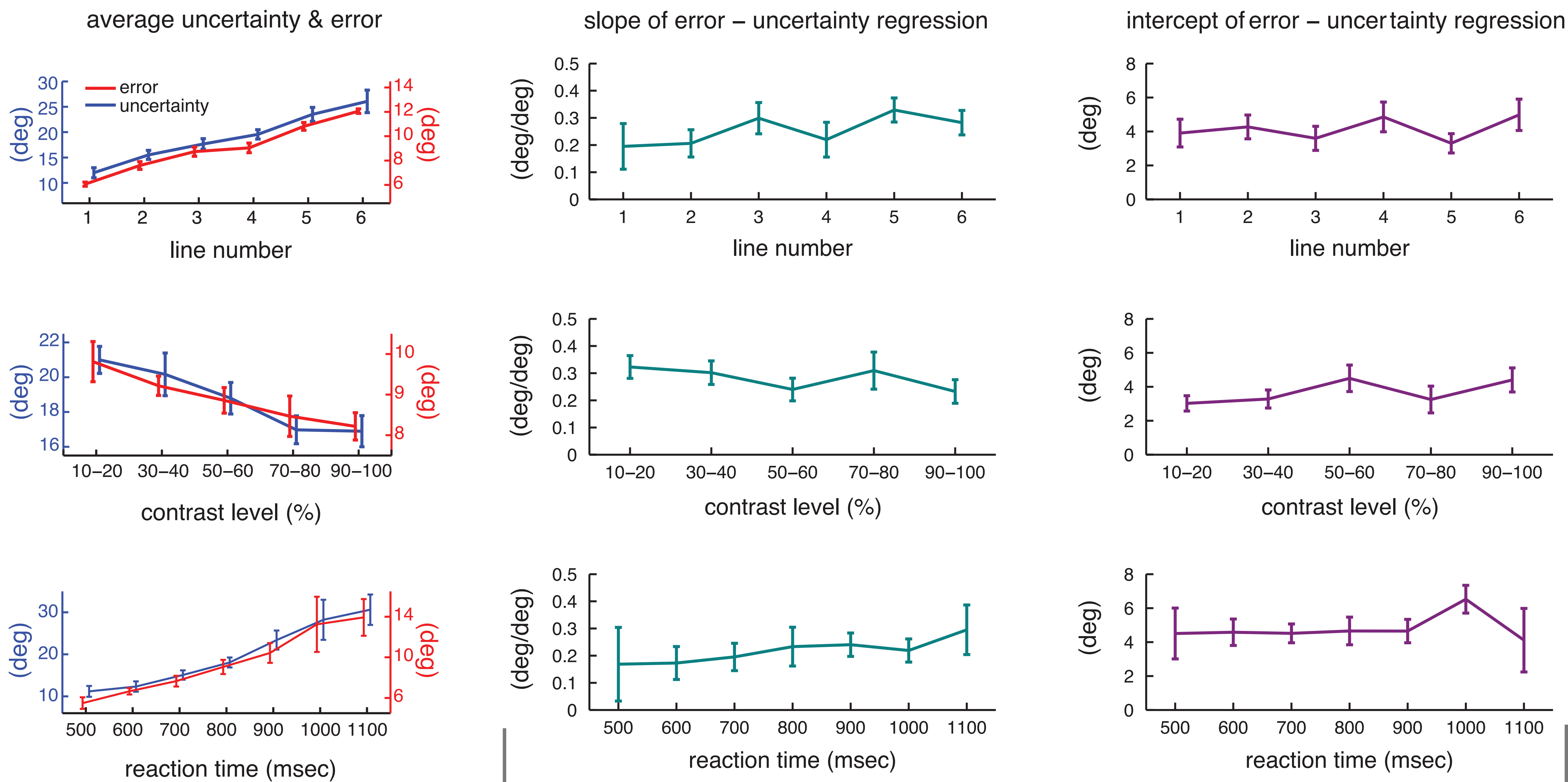
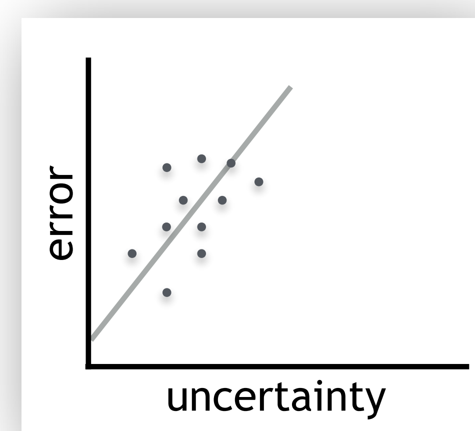


significant

non-significant

a single internal scale for representing uncertainty



EFFECTS OF TASK DIFFICULTY

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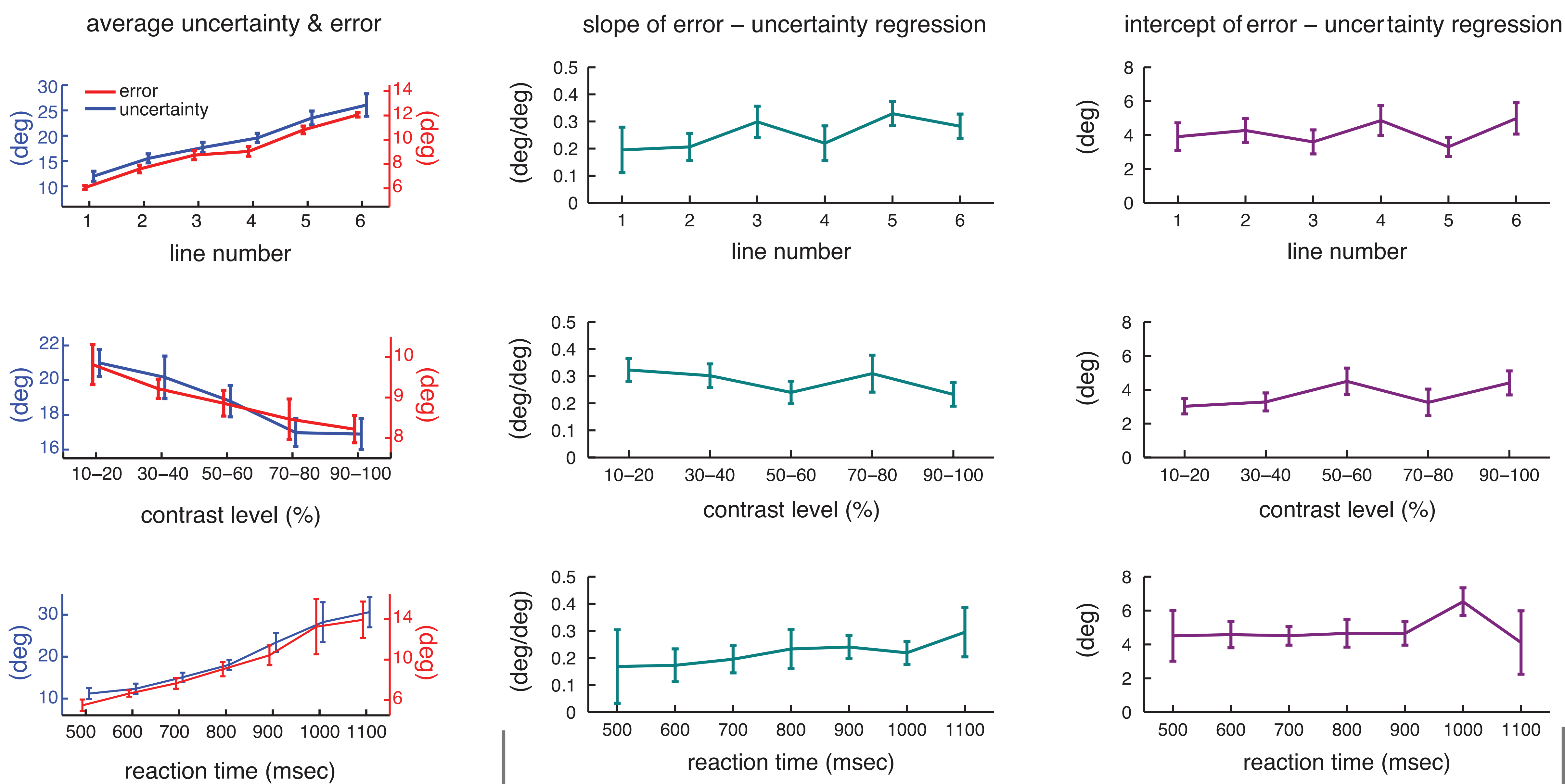

significant

non-significant

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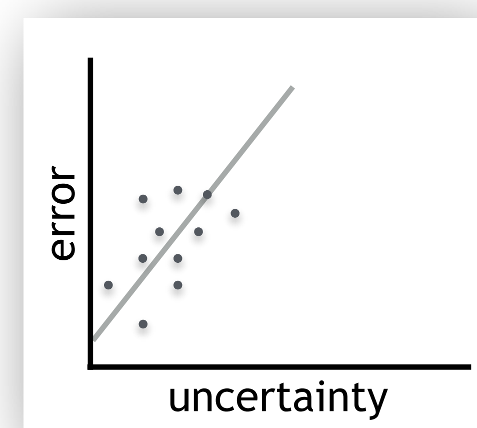
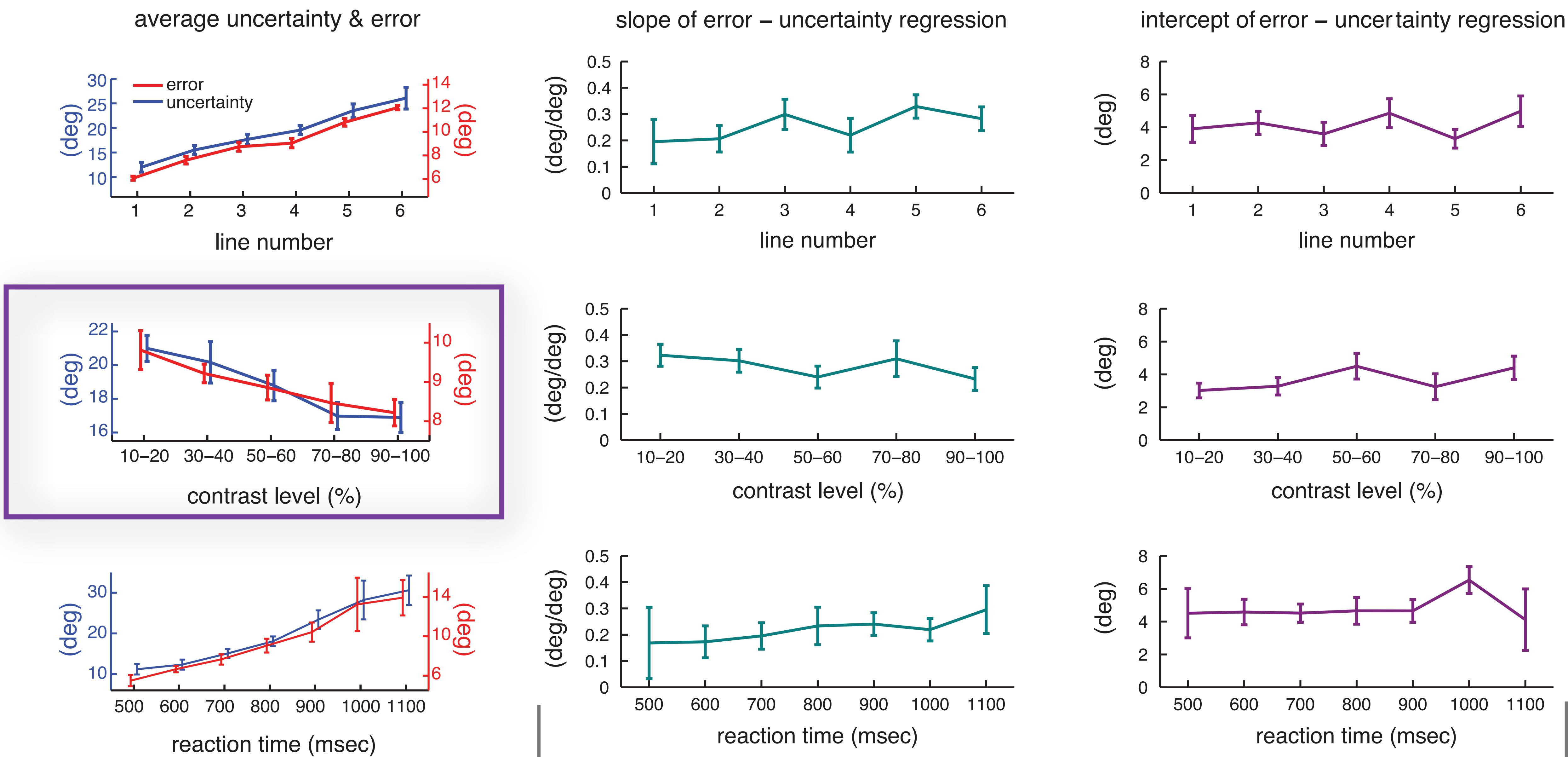


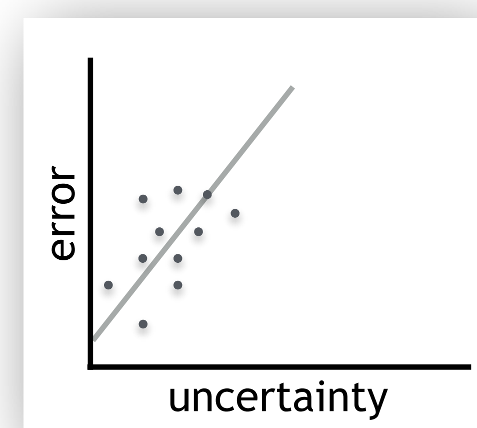
EFFECTS OF TASK DIFFICULTY

Lengyel et al, arXiv 2015



significant

non-significant



a single internal scale for representing uncertainty

a multivariate representation of uncertainty

THEORY: GENERAL SETUP

Lengyel et al, arXiv 2015

THEORY: GENERAL SETUP

Lengyel et al, arXiv 2015

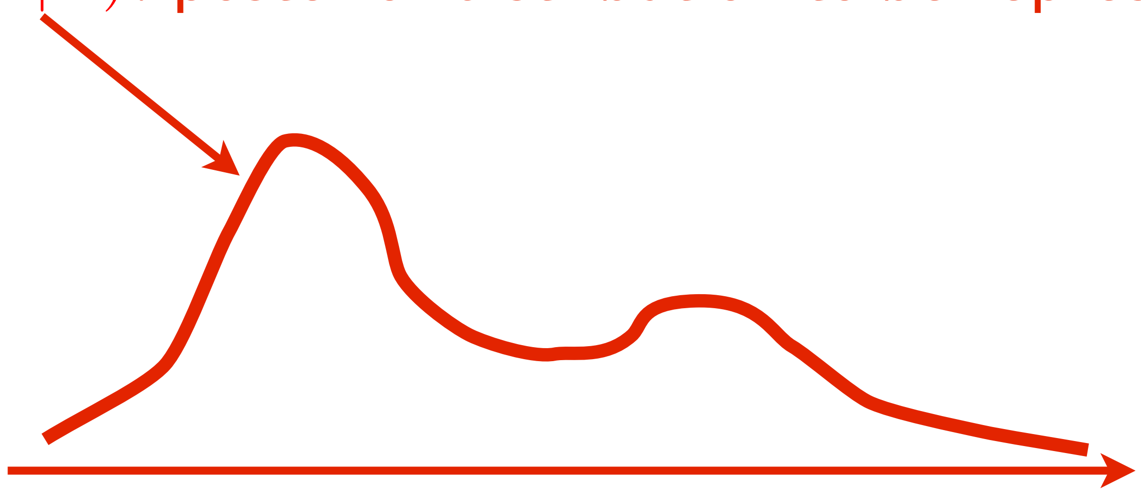
$\mathcal{D}$: observation

THEORY: GENERAL SETUP

Lengyel et al, arXiv 2015

$\mathcal{D}$: observation

$\mathrm{P}(x|\mathcal{D})$: posterior distribution to be represented



x: stimulus to be estimated

THEORY: GENERAL SETUP

Lengyel et al, arXiv 2015

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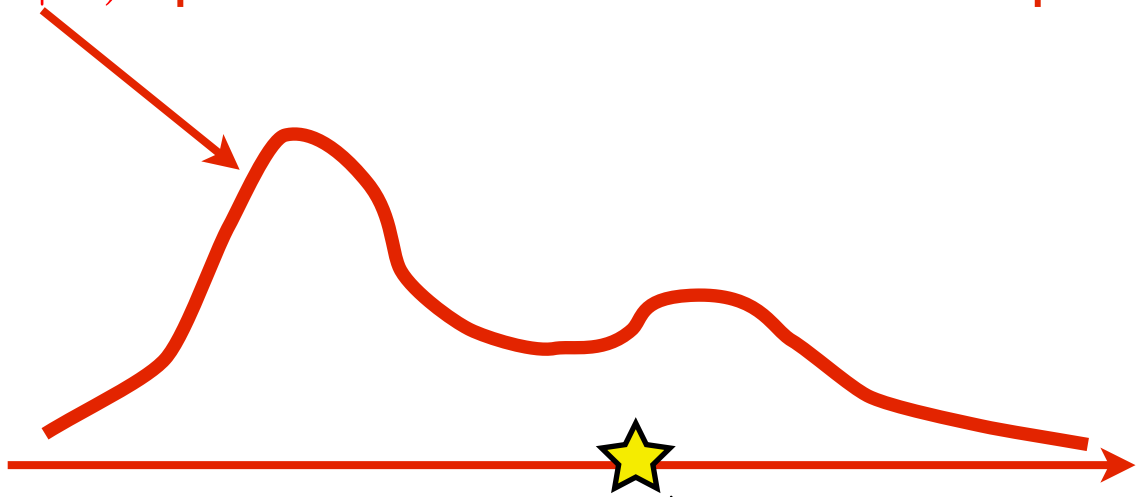


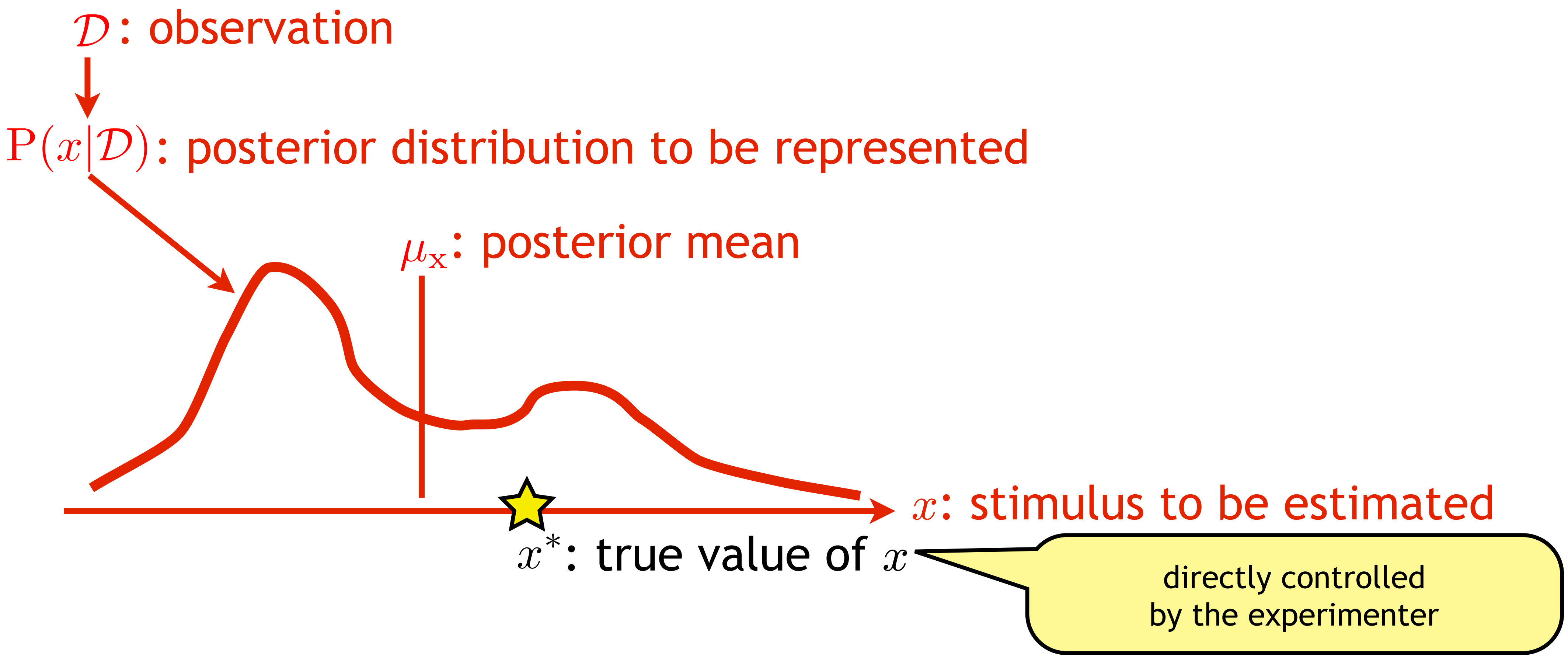
x: stimulus to be estimated

x^*: true value of x

directly controlled
by the experimenter

THEORY: GENERAL SETUP

Lengyel et al, arXiv 2015



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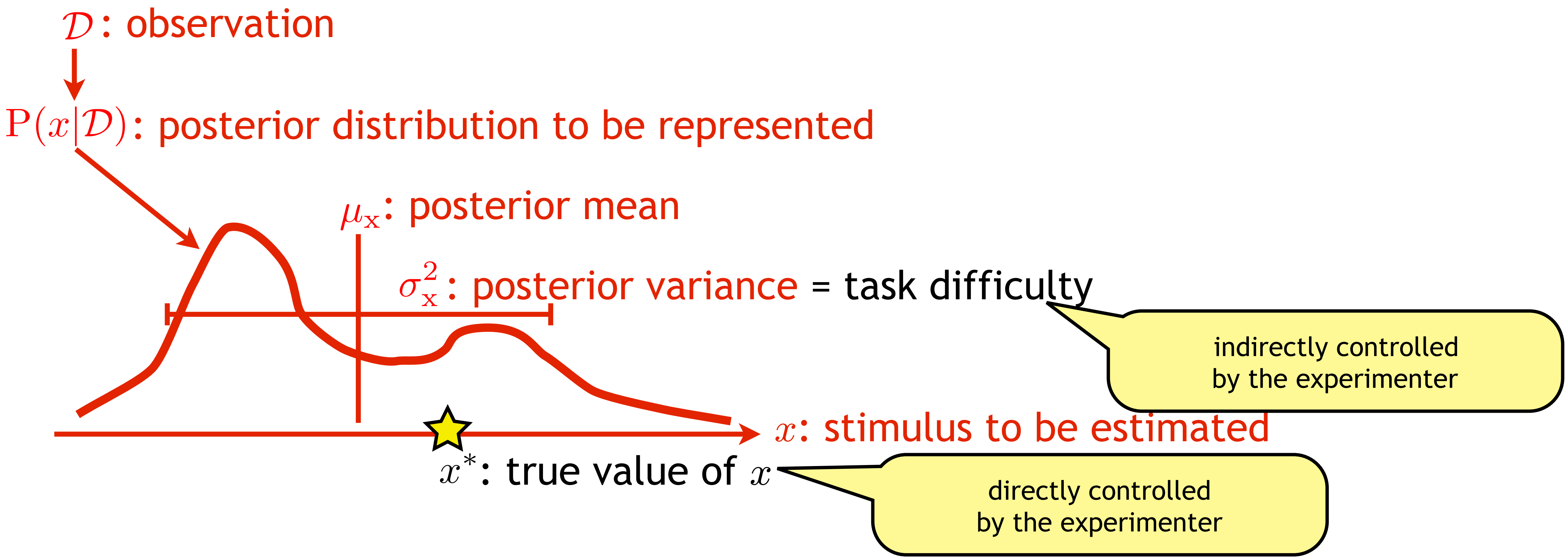

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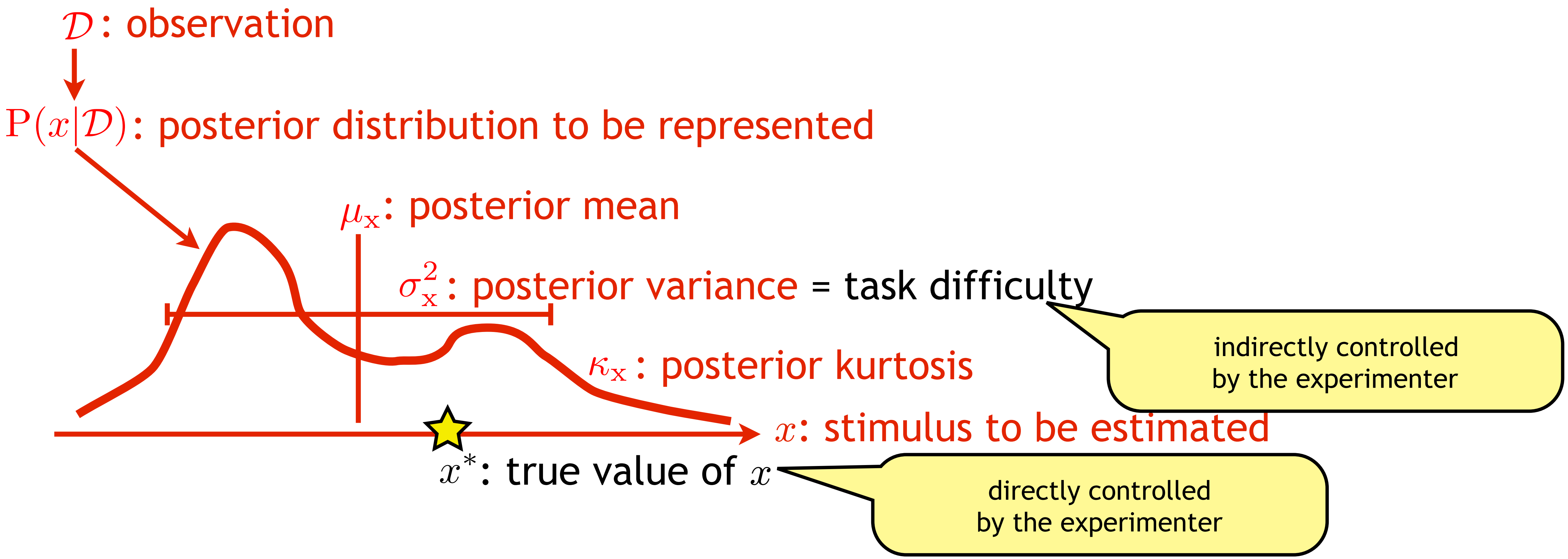
Lengyel et al, arXiv 2015



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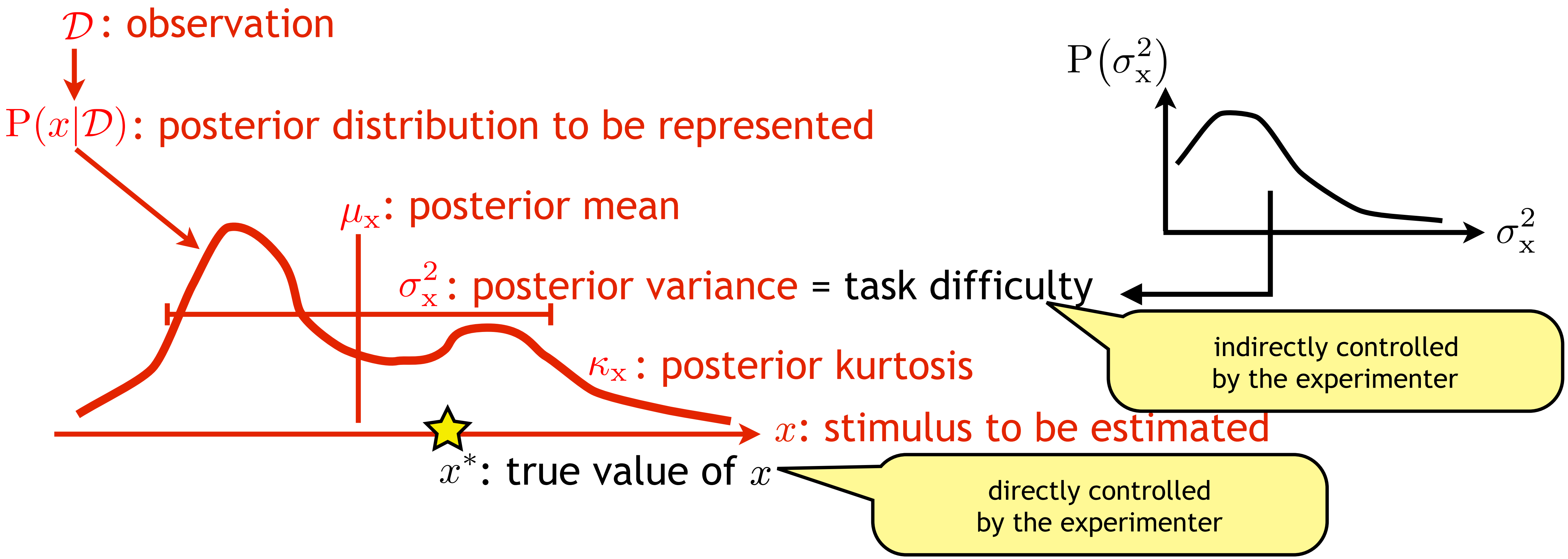
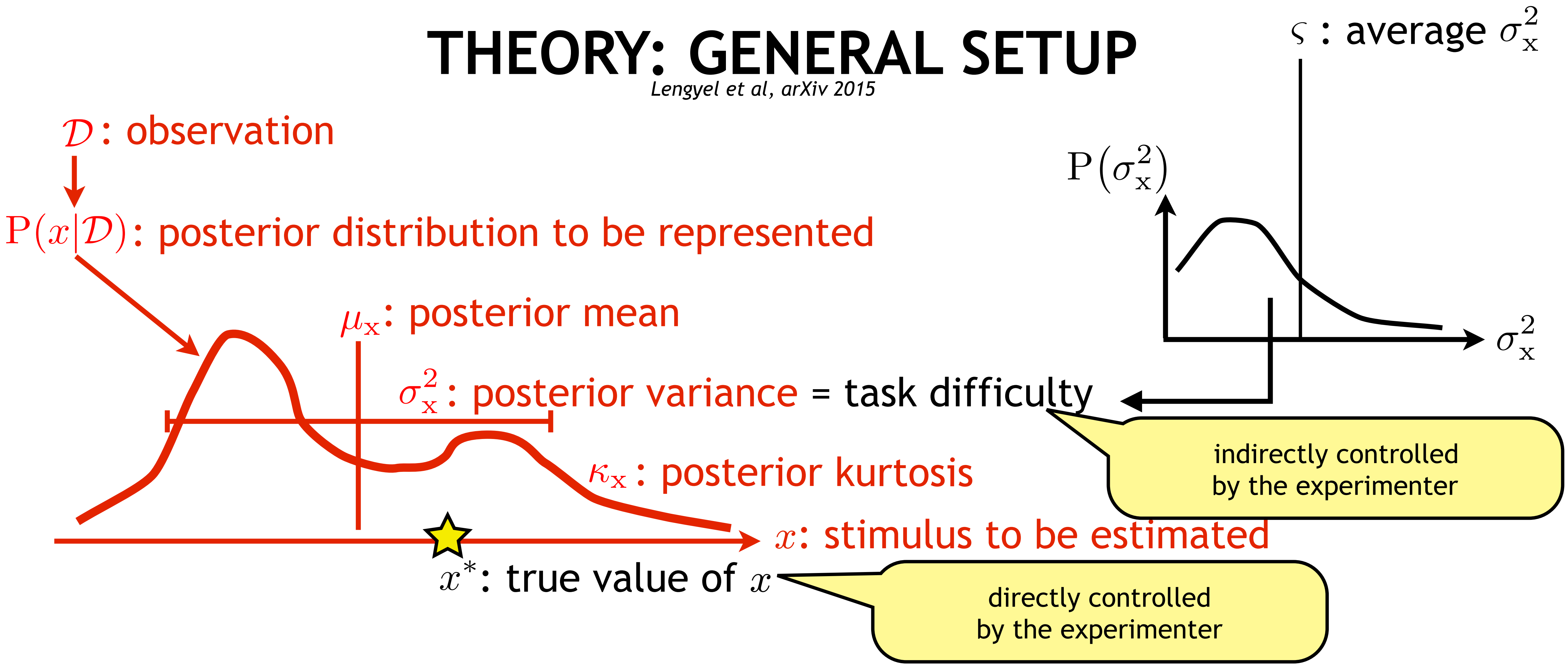


THEORY: GENERAL SETUP

Lengyel et al, arXiv 2015



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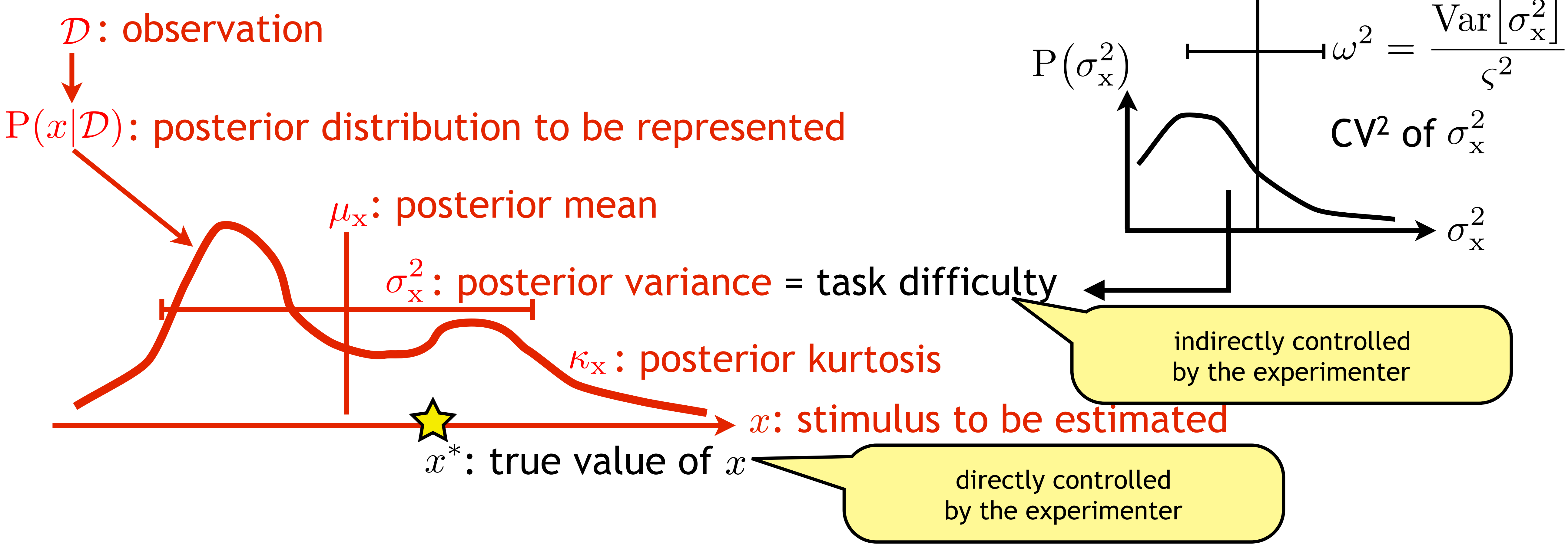

THEORY: GENERAL SETUP

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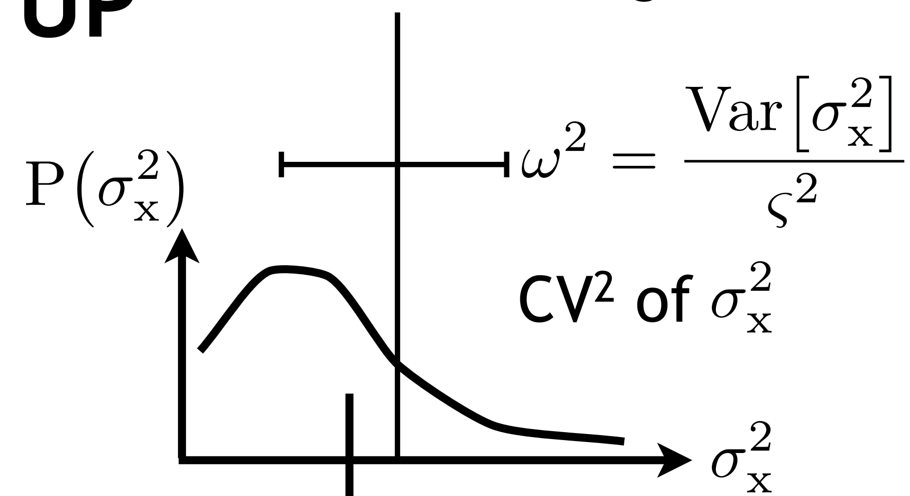
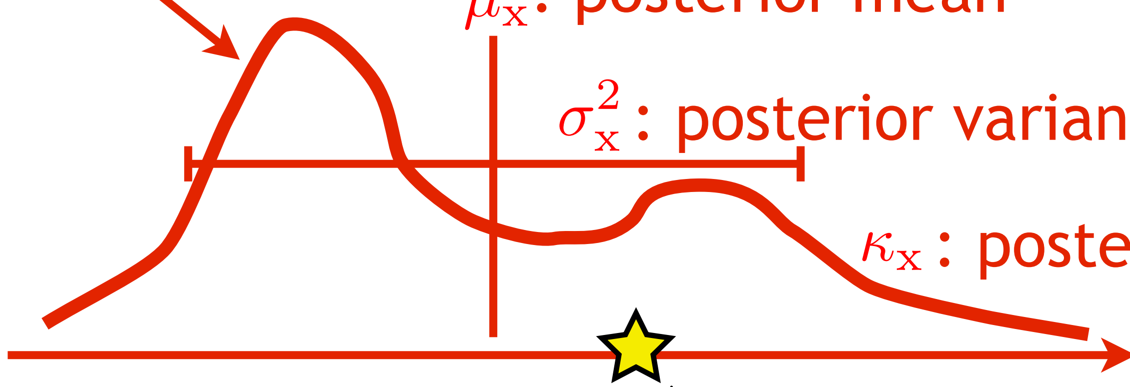


$\mathcal{D}$: observation

$\mathrm{P}(x|\mathcal{D})$: posterior distribution to be represented



indirectly controlled
by the experimenter

directly controlled
by the experimenter

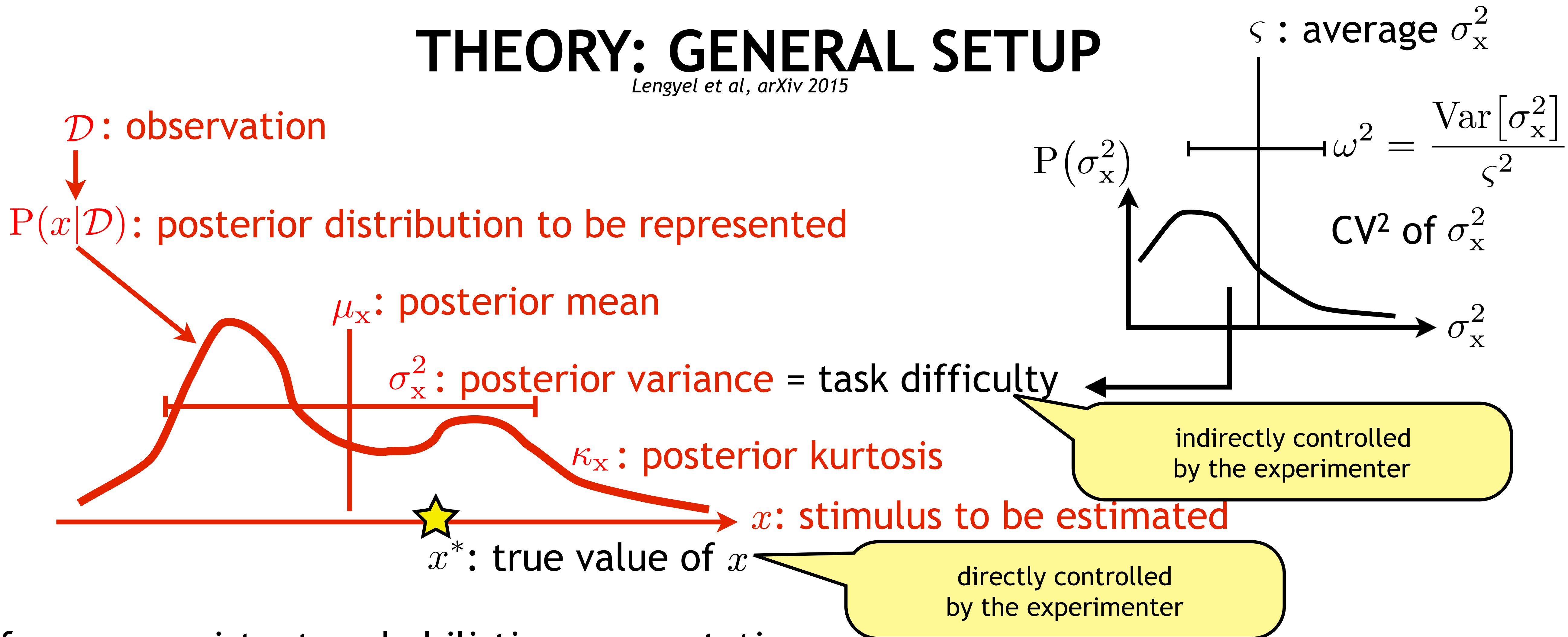
for **any** consistent probabilistic representation:

x^* behaves as if sampled from $\mathrm{P}(x|\mathcal{D})$

$$\begin{aligned}
\mathrm{E}[\mu_{\mathrm{x}}] &= x^* \\
\mathrm{E}\left[(\mu_{\mathrm{x}} - x^*)^2\right] &= \sigma_{\mathrm{x}}^2 \\
\mathrm{E}\left[(\mu_{\mathrm{x}} - x^*)^4\right] &= (\kappa_{\mathrm{x}} + 3)\,\sigma_{\mathrm{x}}^4
\end{aligned}$$

THEORY: GENERAL SETUP

Lengyel et al, arXiv 2015



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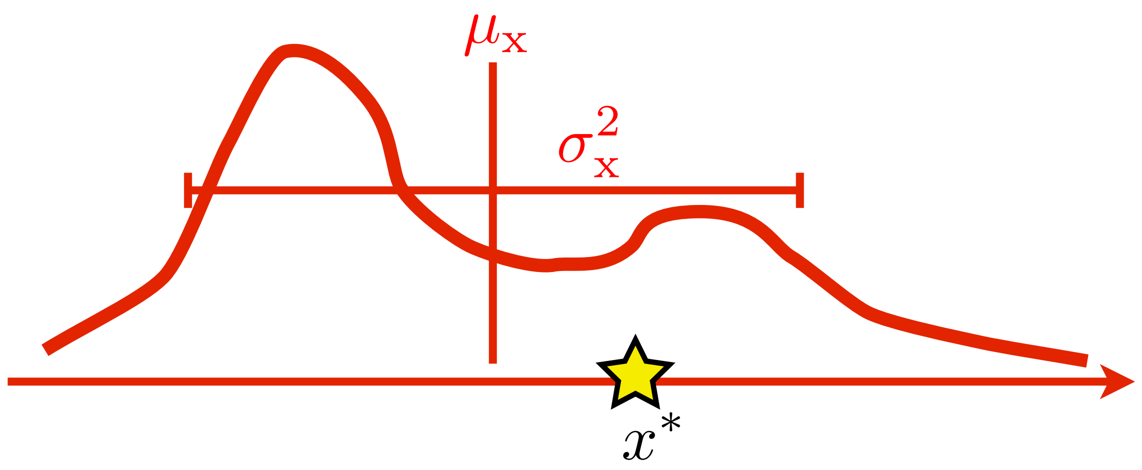
\mathrm{E}\left[(\mu_\mathrm{x} - x^*)^4\right] &= (\kappa_\mathrm{x} + 3)\,\sigma_\mathrm{x}^4
\end{aligned}$$

no parametric assumptions about
$\mathrm{P}(x|\mathcal{D})$ or $\mathrm{P}\left(\sigma_\mathrm{x}^2\right)$

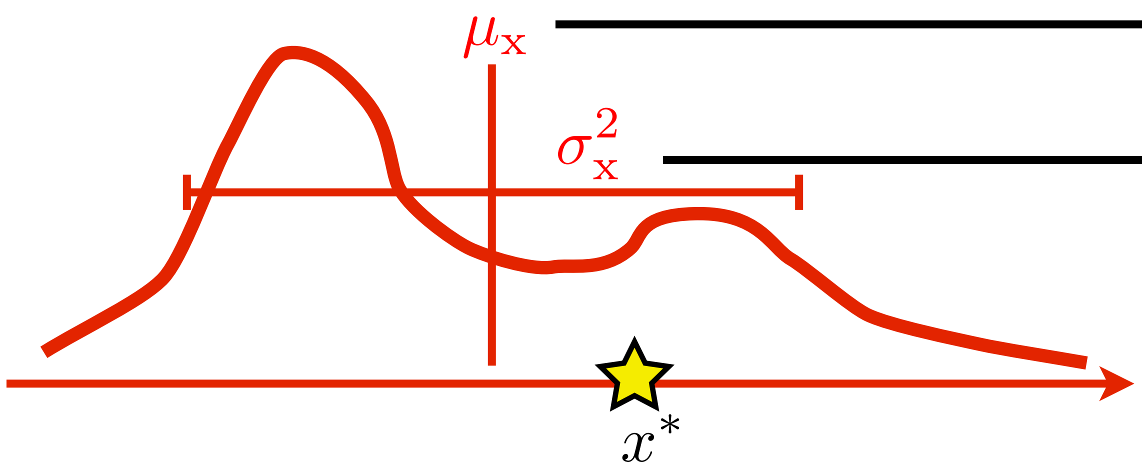
EXACT, STATIC PROBABILISTIC REPRESENTATION

Lengyel et al, arXiv 2015



EXACT, STATIC PROBABILISTIC REPRESENTATION

Lengyel et al, arXiv 2015



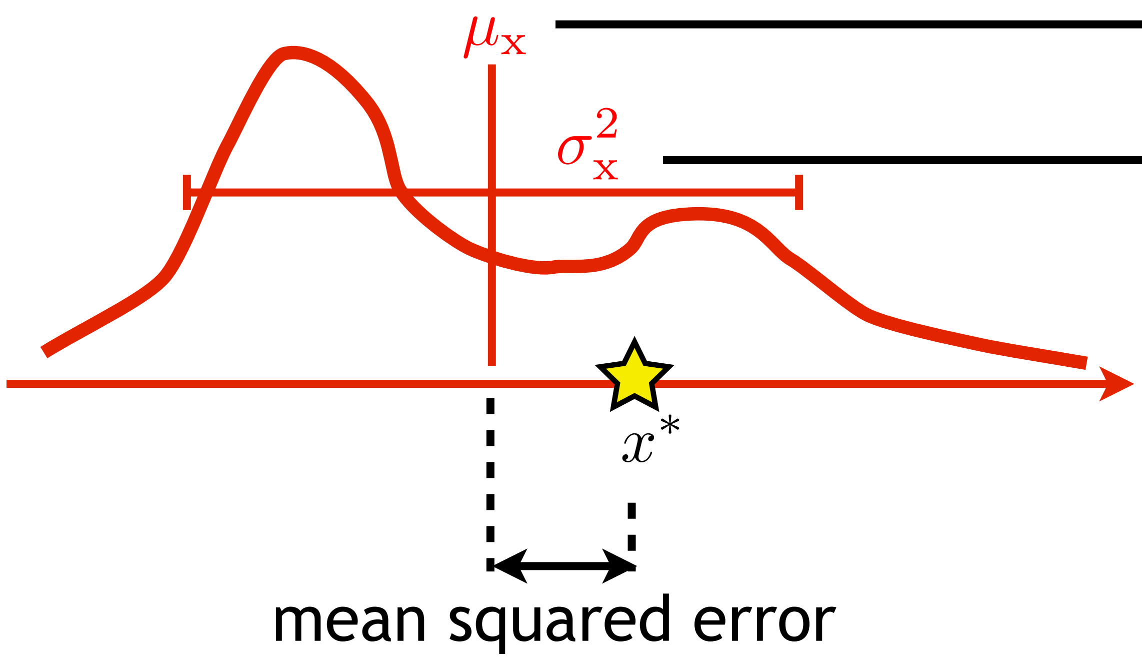
behavioral reports

stimulus estimate

uncertainty

EXACT, STATIC PROBABILISTIC REPRESENTATION

Lengyel et al, arXiv 2015



$$\varepsilon^2 = \varsigma$$

behavioral reports

stimulus estimate

uncertainty

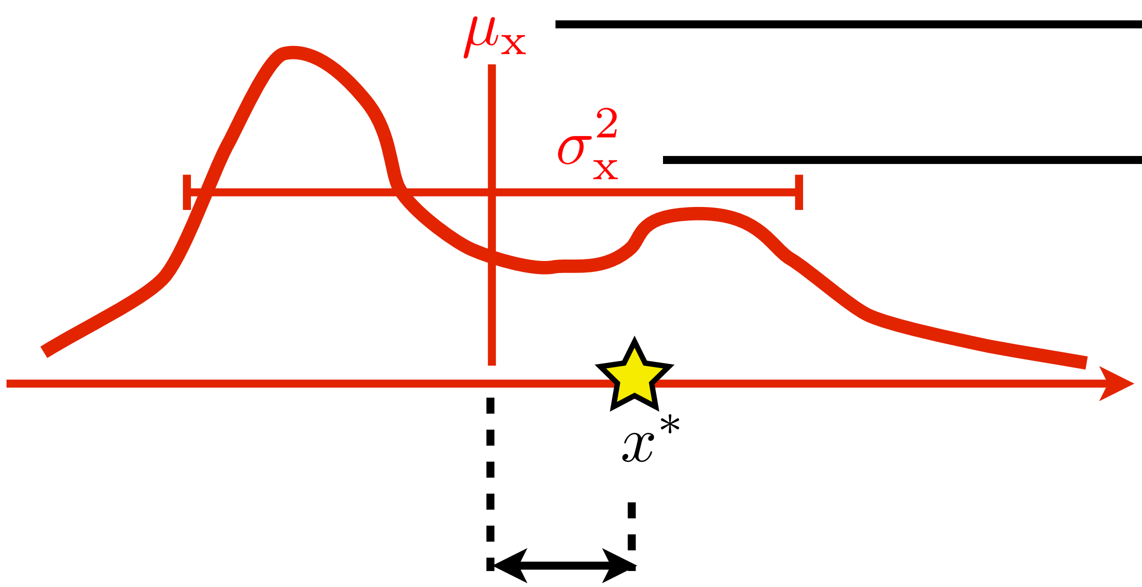
EXACT, STATIC PROBABILISTIC REPRESENTATION

Lengyel et al, arXiv 2015

behavioral reports

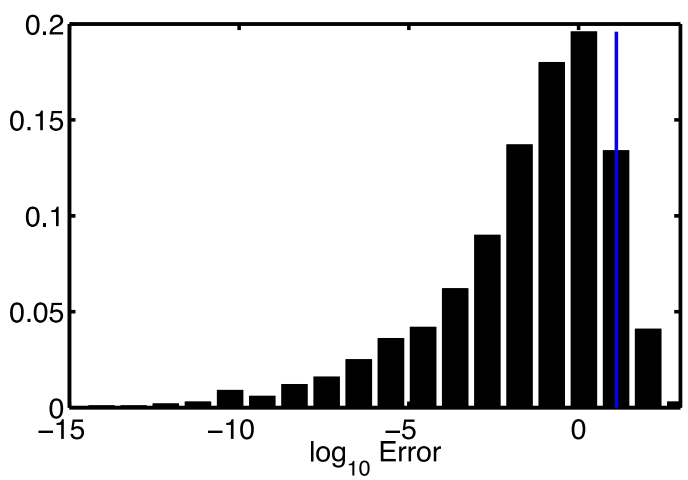
stimulus estimate

uncertainty



mean squared error

$$\varepsilon^2 = \varsigma$$



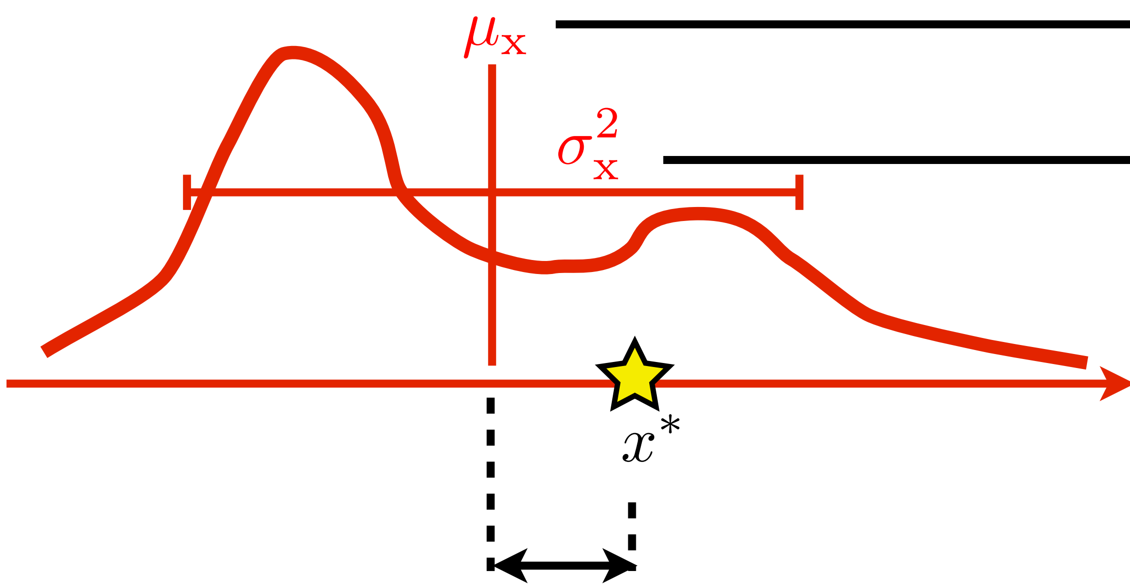
EXACT, STATIC PROBABILISTIC REPRESENTATION

Lengyel et al, arXiv 2015

behavioral reports

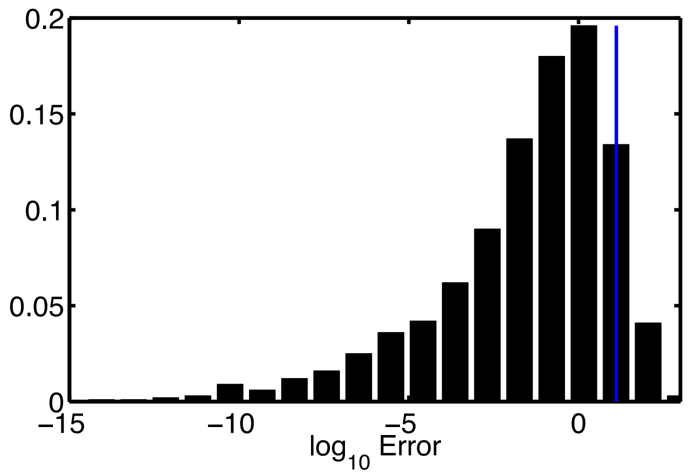
stimulus estimate

uncertainty



mean squared error

$$\varepsilon^2 = \varsigma$$



error-uncertainty correlation

$$\varrho = \frac{1}{\sqrt{\kappa_{\mathrm{x}} + 3 + (\kappa_{\mathrm{x}} + 2)\ \omega^{-2}}}$$

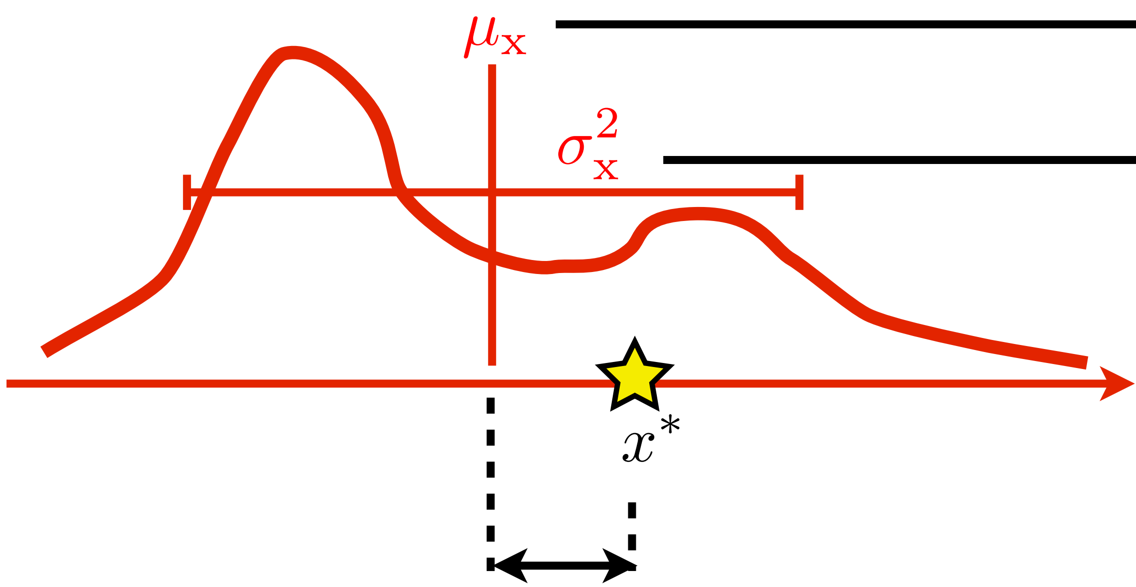
EXACT, STATIC PROBABILISTIC REPRESENTATION

Lengyel et al, arXiv 2015

behavioral reports

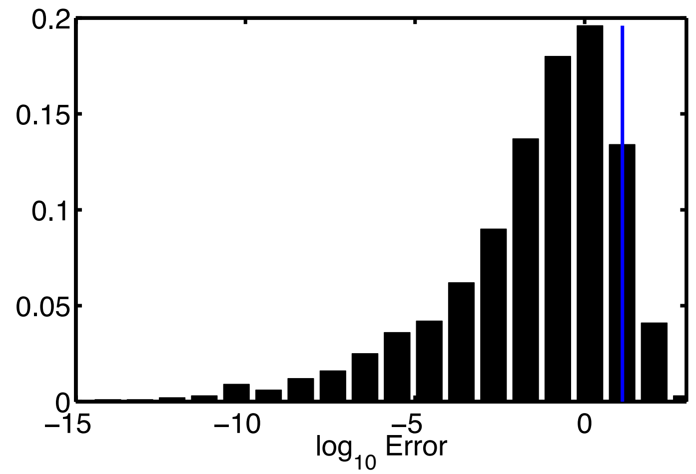
stimulus estimate

uncertainty



mean squared error

$$\varepsilon^2 = \varsigma$$



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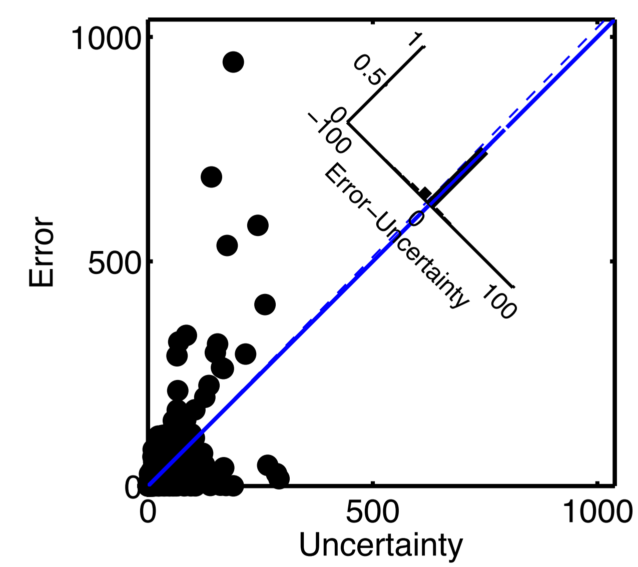

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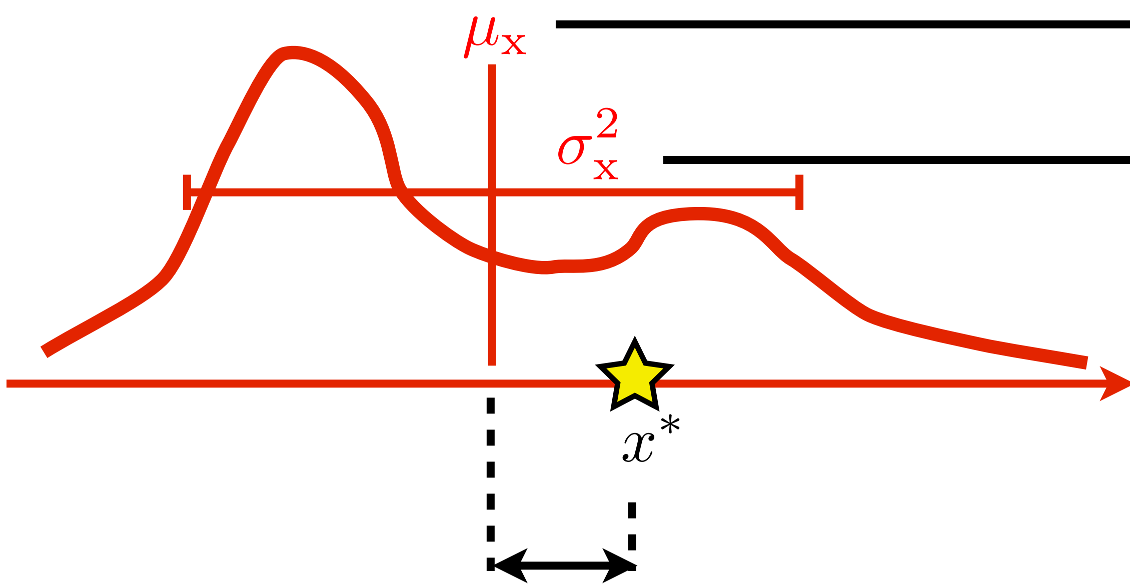
Lengyel et al, arXiv 2015



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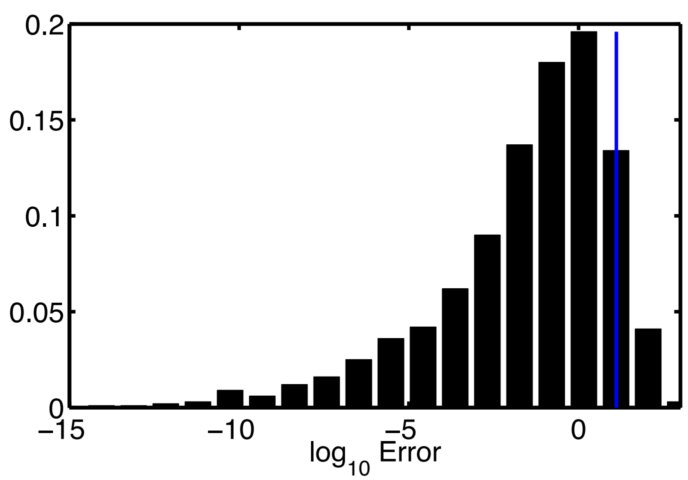

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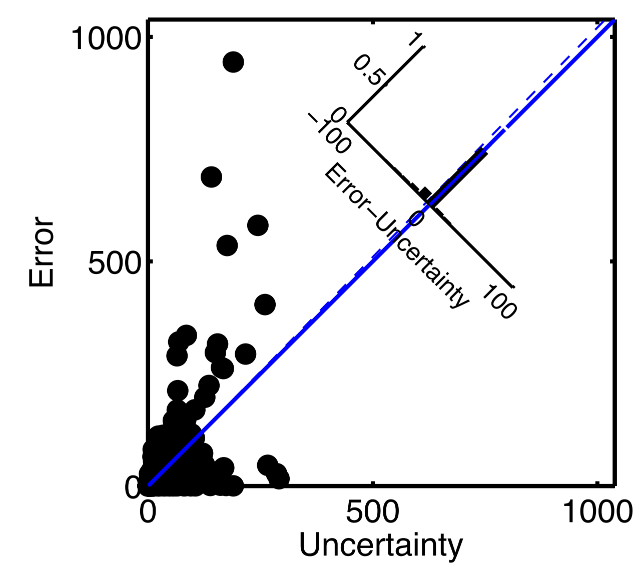

$\varrho < 1$ even for an exact representation*

need trials with varying task difficulty ($\omega^2 > 0$)

*except when the posterior is symmetric Bernoulli, ie $\kappa_{\mathrm{x}} = -2$ → $\varrho = 1$

EVIDENCE INTEGRATION
EVIDENCE INTEGRATION

Lengyel et al, arXiv 2015

EVIDENCE INTEGRATION

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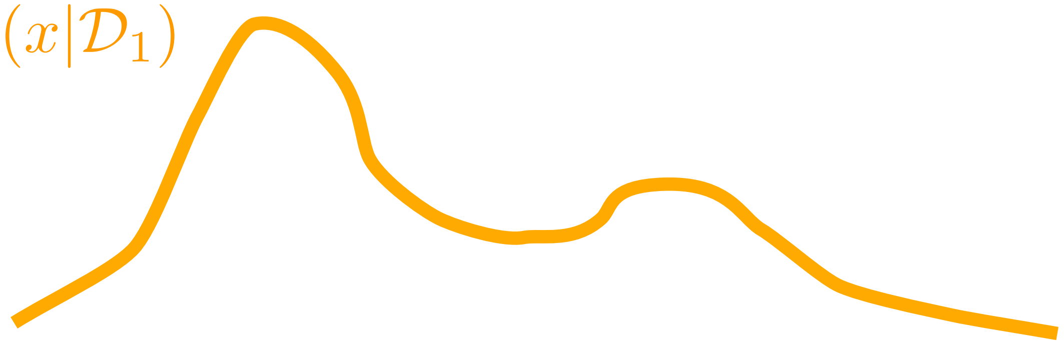
Lengyel et al, arXiv 2015

$\mathcal{D}_1 \longrightarrow \mathrm{P}(x|\mathcal{D}_1)$



EVIDENCE INTEGRATION

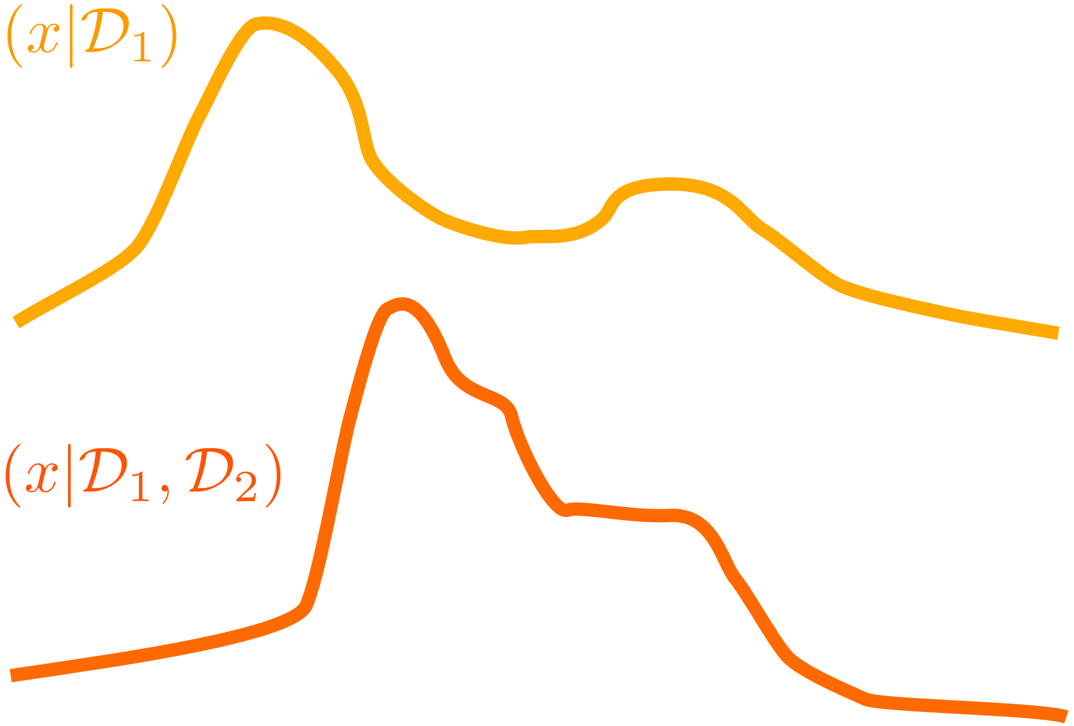
EVIDENCE INTEGRATION

Lengyel et al, arXiv 2015

$\mathcal{D}_1 \rightarrow \mathrm{P}(x|\mathcal{D}_1)$

$\downarrow$

$\mathcal{D}_2 \rightarrow \mathrm{P}(x|\mathcal{D}_1, \mathcal{D}_2)$



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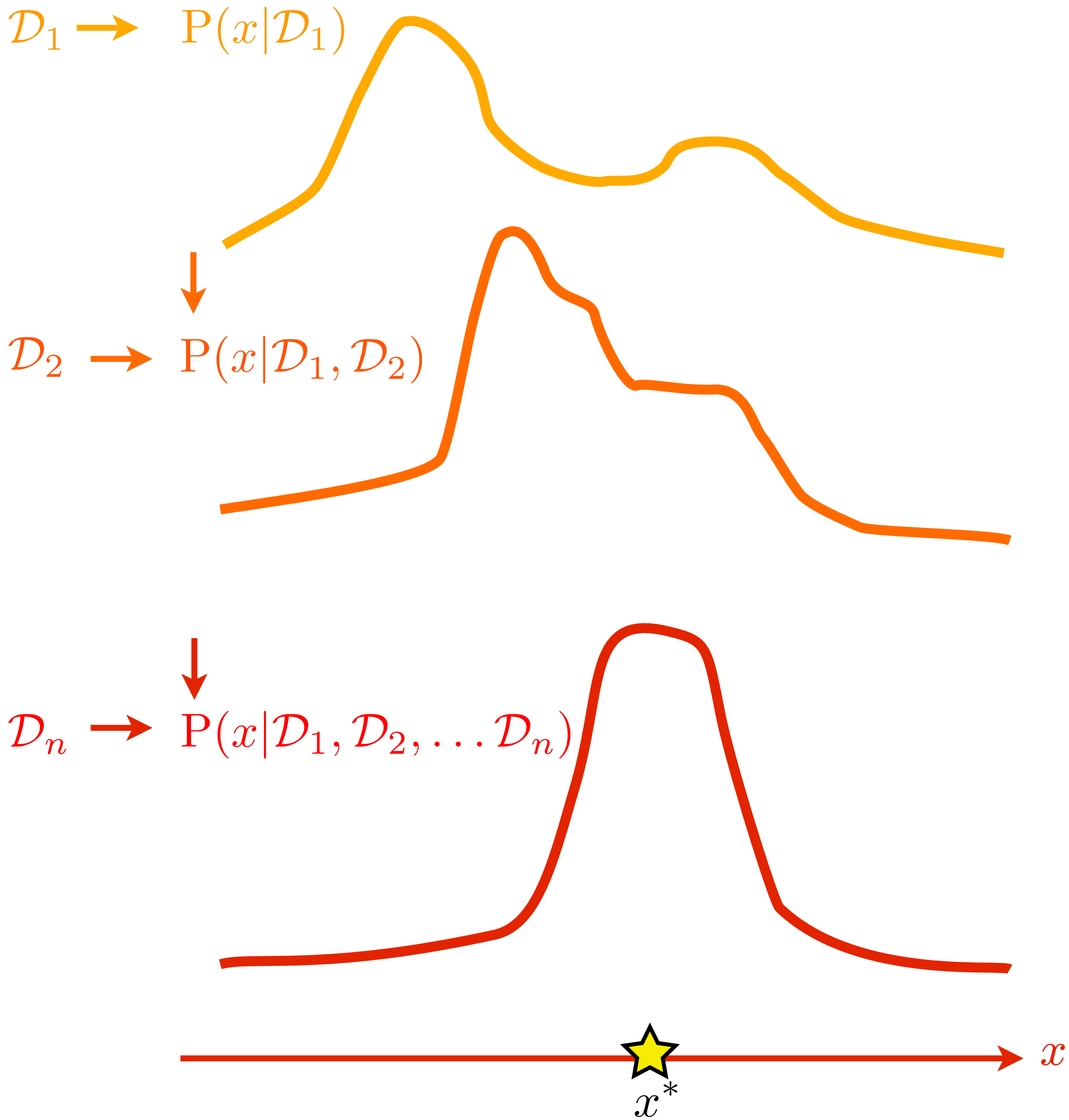

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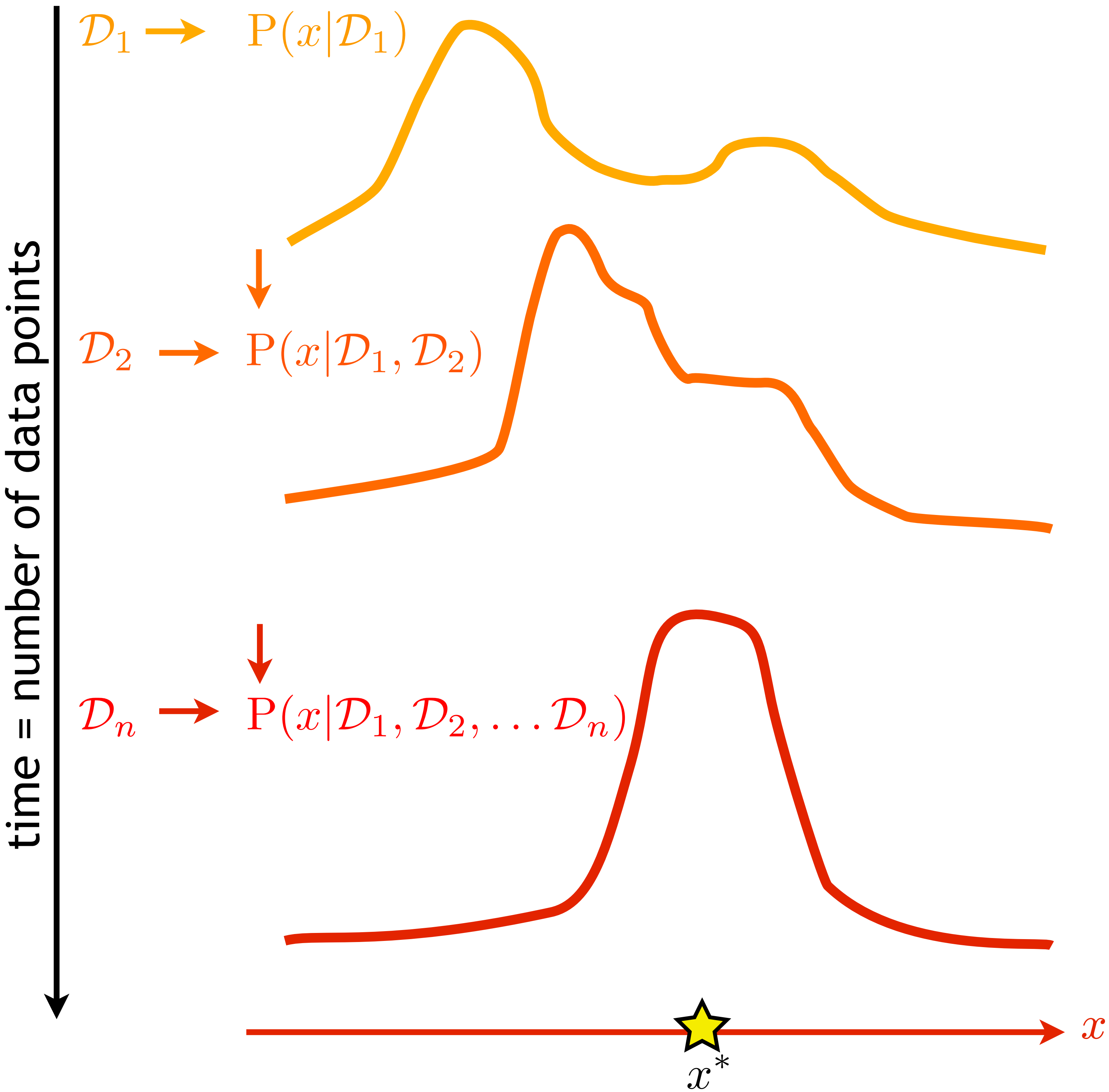

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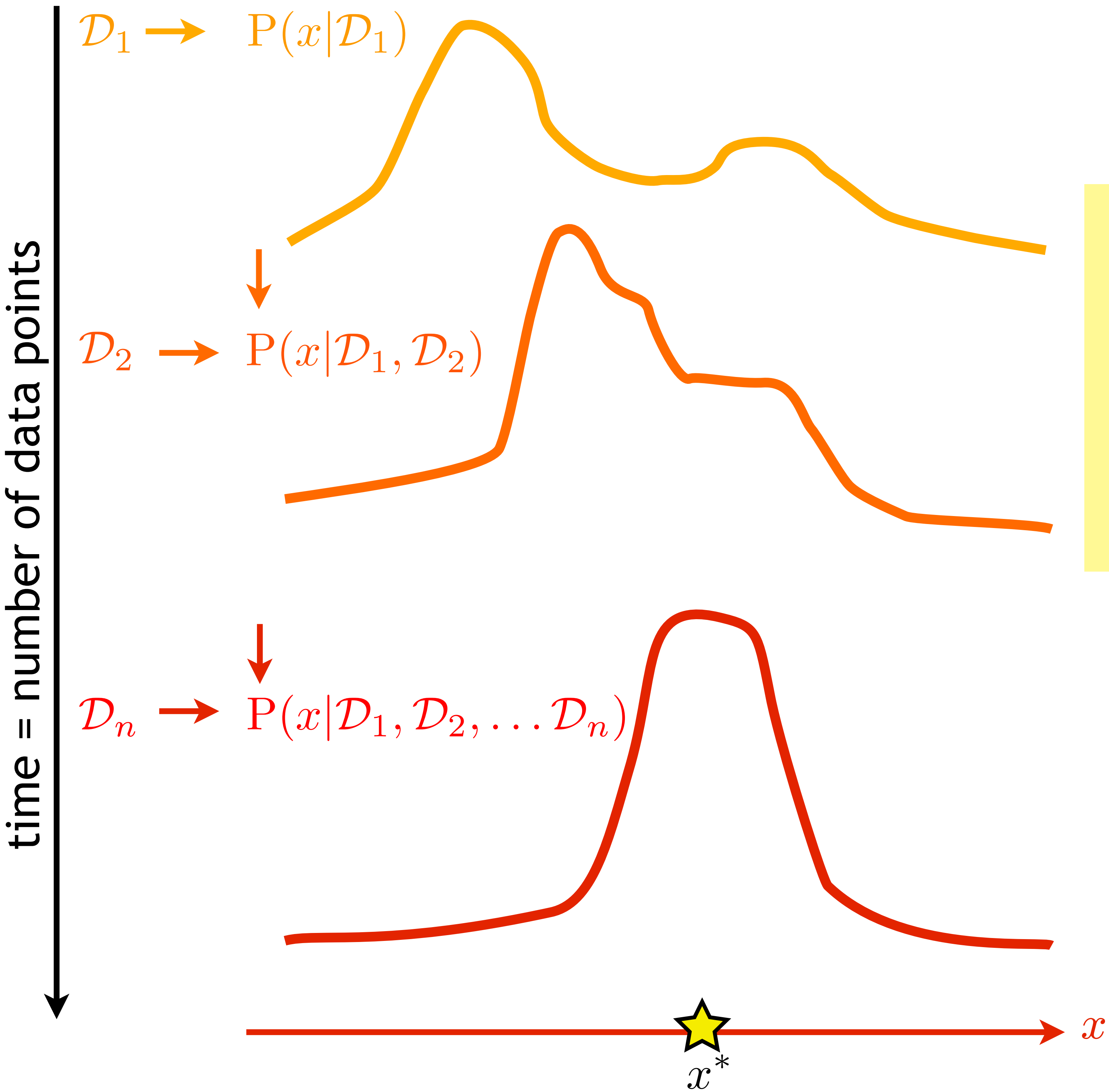


assumptions:

with time, posterior becomes

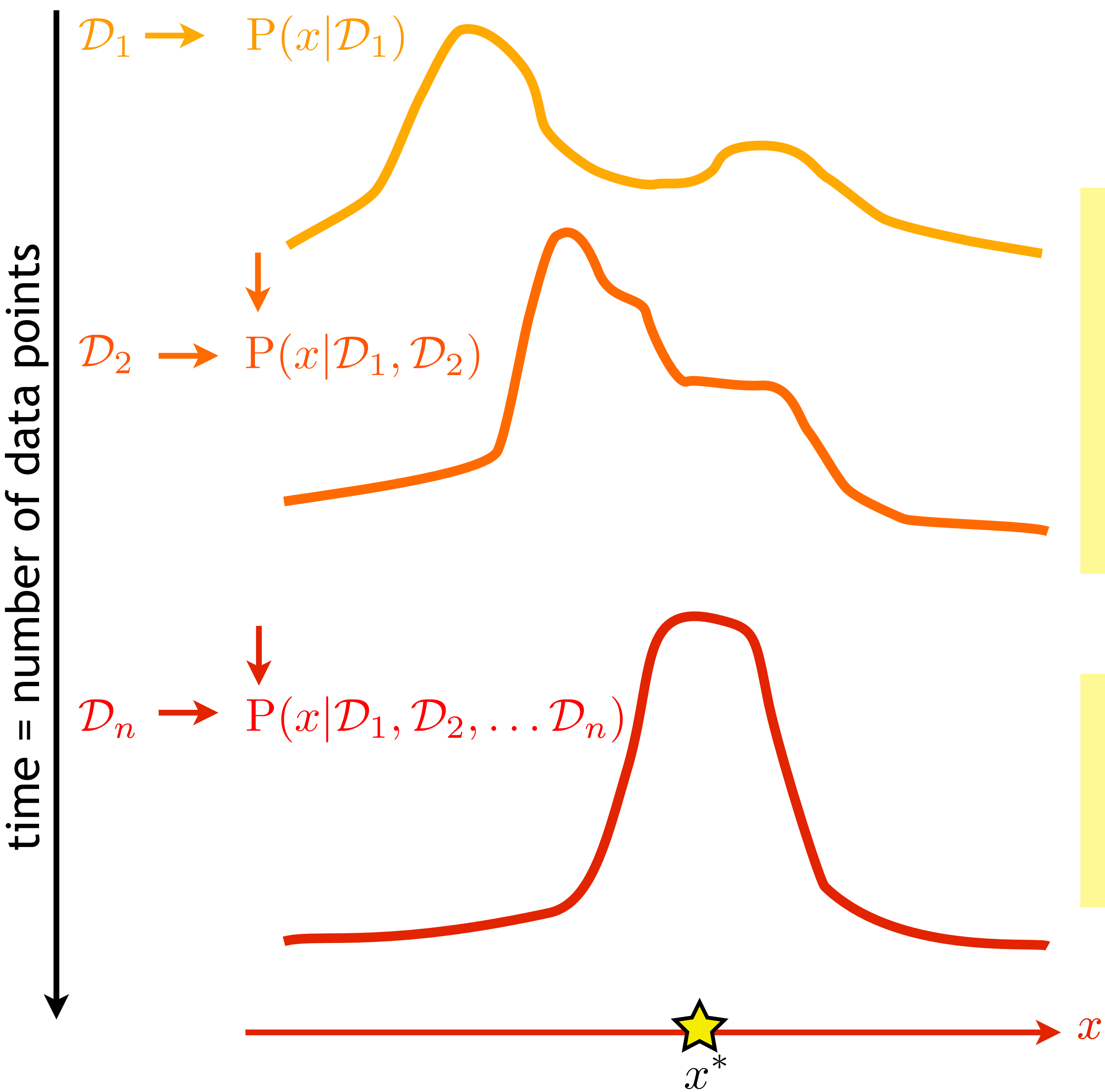
– narrower and more centred on x^*

$\sigma_\mathrm{x}^2 \propto 1/n \leftrightarrow \varsigma \propto 1/n$

– more Gaussian-like $\kappa_\mathrm{x} \propto 1/n$

EVIDENCE INTEGRATION

Lengyel et al, arXiv 2015



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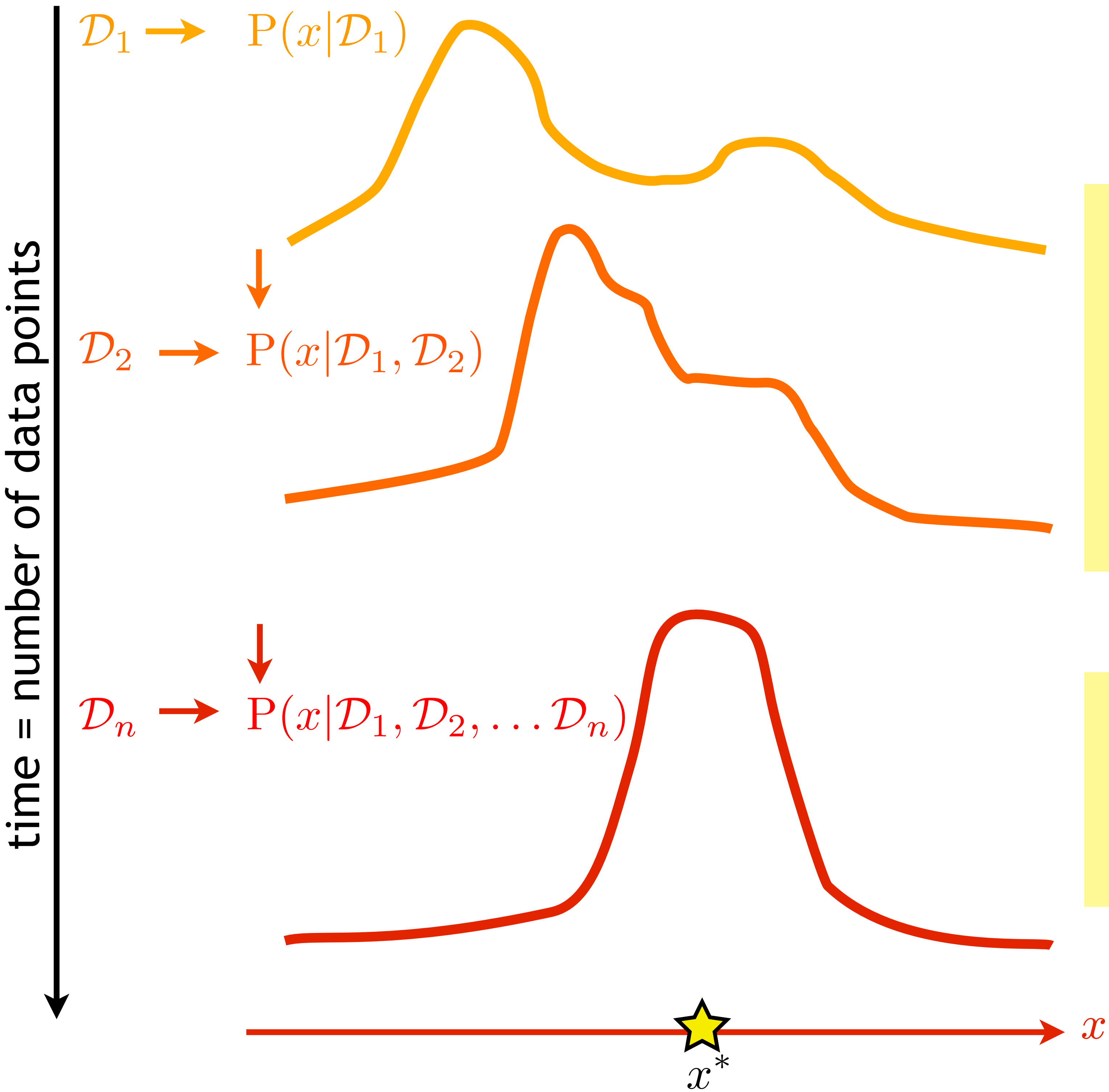
behavioral reports

– stimulus estimate μ_{x}

– uncertainty σ_{x}^2

EVIDENCE INTEGRATION + BEHAVIOURAL NOISE

Lengyel et al, arXiv 2015



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$\sigma^2_{\mathrm{x}} \propto 1/n \leftrightarrow \varsigma \propto 1/n$

– more Gaussian-like $\kappa_{\mathrm{x}} \propto 1/n$

behavioral reports are noisy

– stimulus estimate $\mu_{\mathrm{x}} + \mathrm{noise}_\mu$

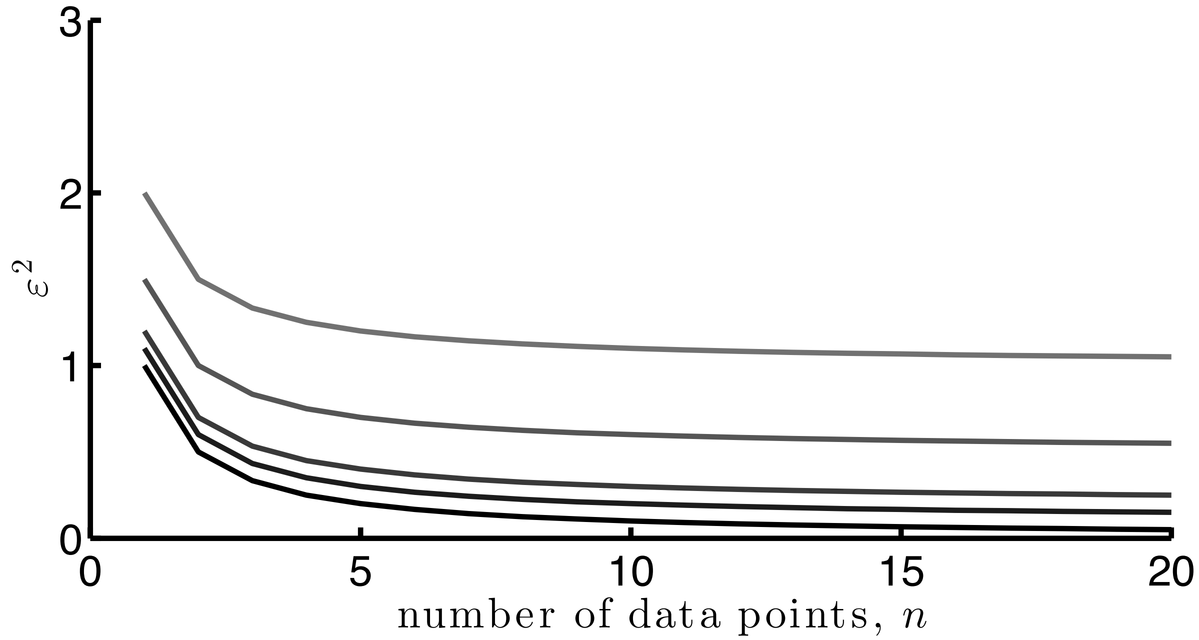
– uncertainty $\sigma^2_{\mathrm{x}} + \mathrm{noise}_{\sigma^2}$

EVIDENCE INTEGRATION + NOISE

EVIDENCE INTEGRATION + NOISE

mean squared error

$$\varepsilon^2 = \varsigma/n + \epsilon_1$$



EVIDENCE INTEGRATION + NOISE

mean squared error

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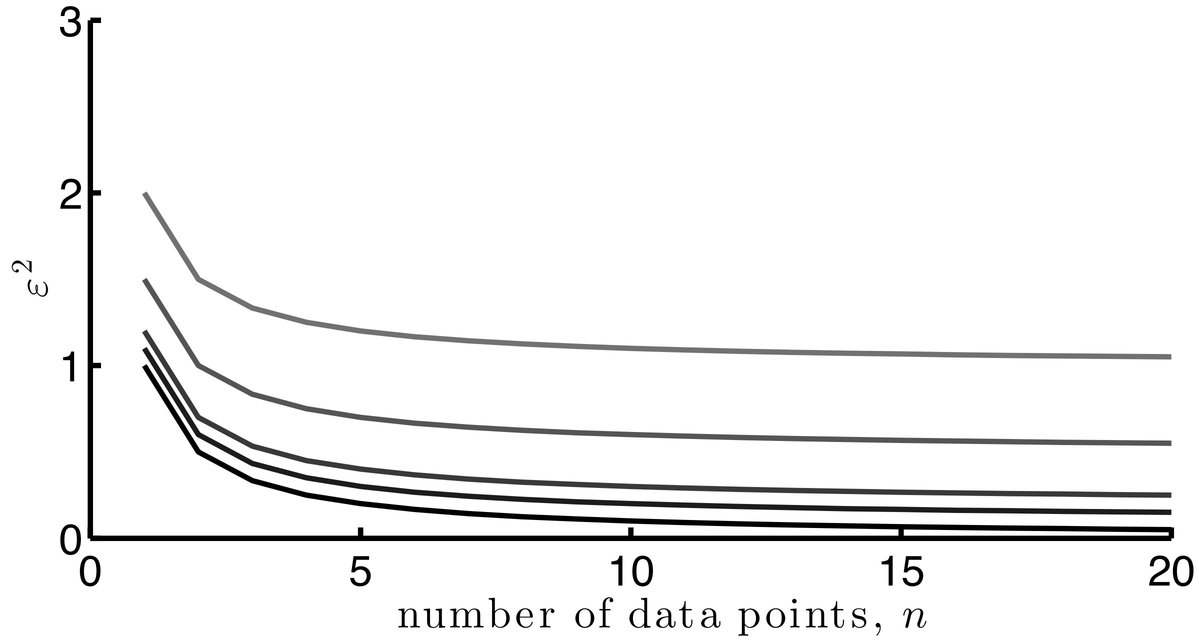
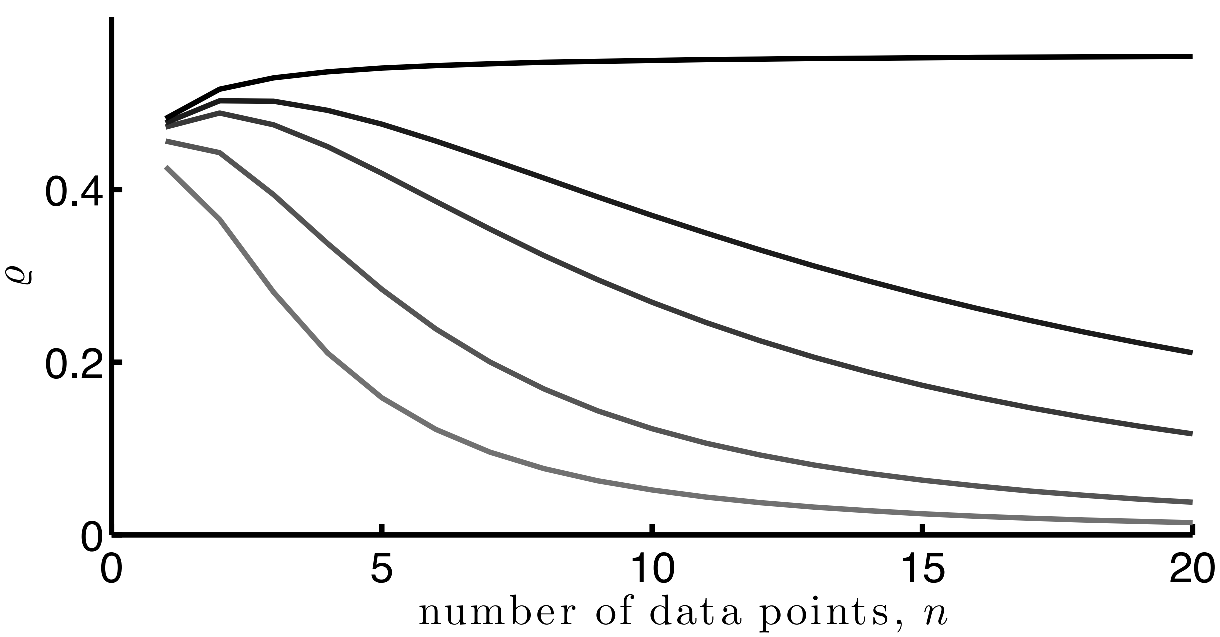


error-uncertainty correlation

$$\varrho = \frac{1}{\sqrt{\left(\kappa_\mathrm{x}/n + 3 + (\kappa_\mathrm{x}/n + 2)\,\omega^{-2} + n^2\,\epsilon_2 + \epsilon_3\right)\,\left(1 + n^2\,\epsilon_4\right)}}$$

$\text{noise}_\mu : \ \epsilon_1, \epsilon_2, \epsilon_3$

$\text{noise}_{\sigma^2} : \epsilon_4$



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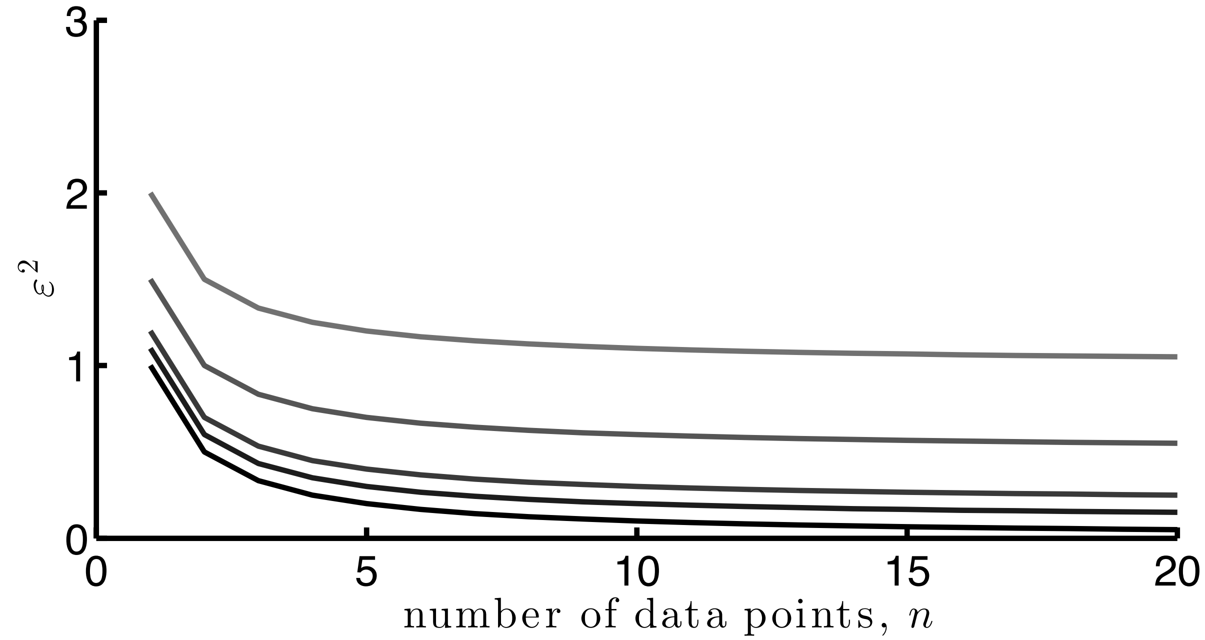


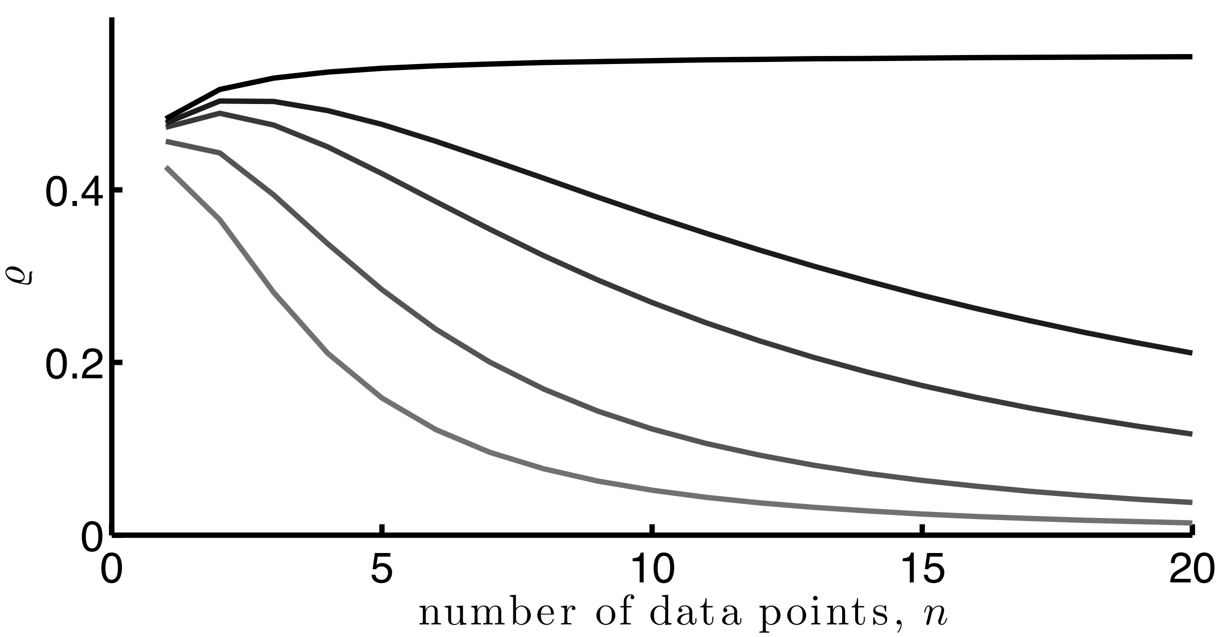
small noise
error → zero

error-uncertainty correlation

$$\varrho = \frac{1}{\sqrt{(\kappa_x/n + 3 + (\kappa_x/n + 2)\,\omega^{-2} + n^2\,\epsilon_2 + \epsilon_3)\;(1 + n^2\,\epsilon_4)}}$$

noise$_\mu$: $\epsilon_1, \epsilon_2, \epsilon_3$

noise$_{\sigma^2}$: ϵ_4



small noise
correlation can increase
(faster than error decreases)

EVIDENCE INTEGRATION + NOISE

mean squared error

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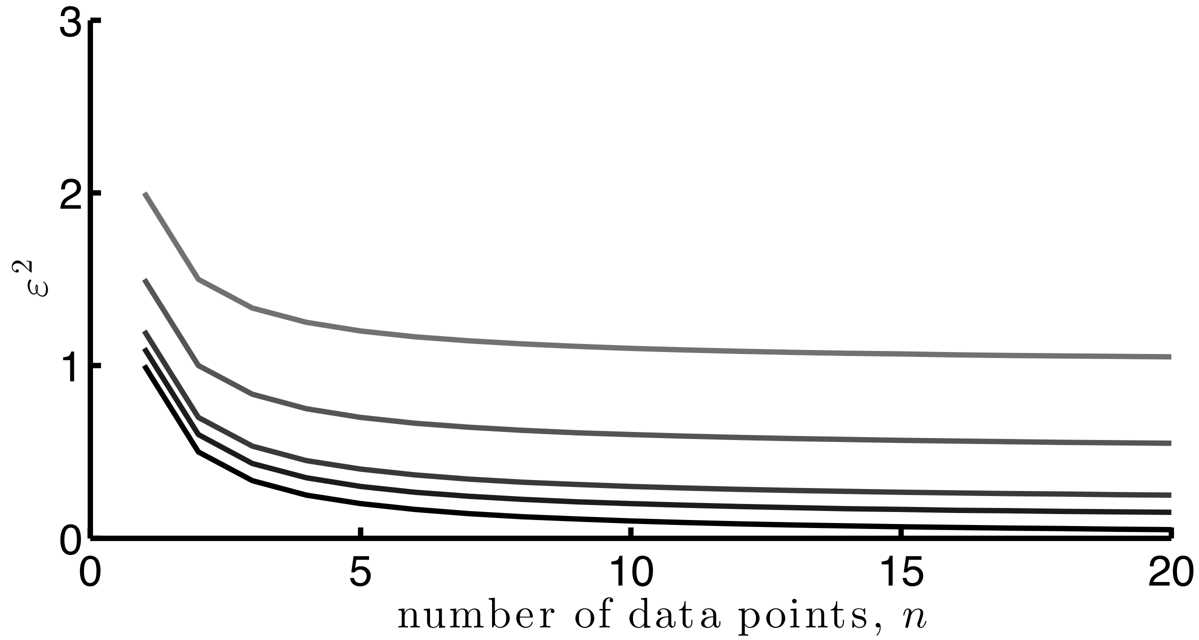

large noise
error → asymptote

small noise
error → zero

error-uncertainty correlation

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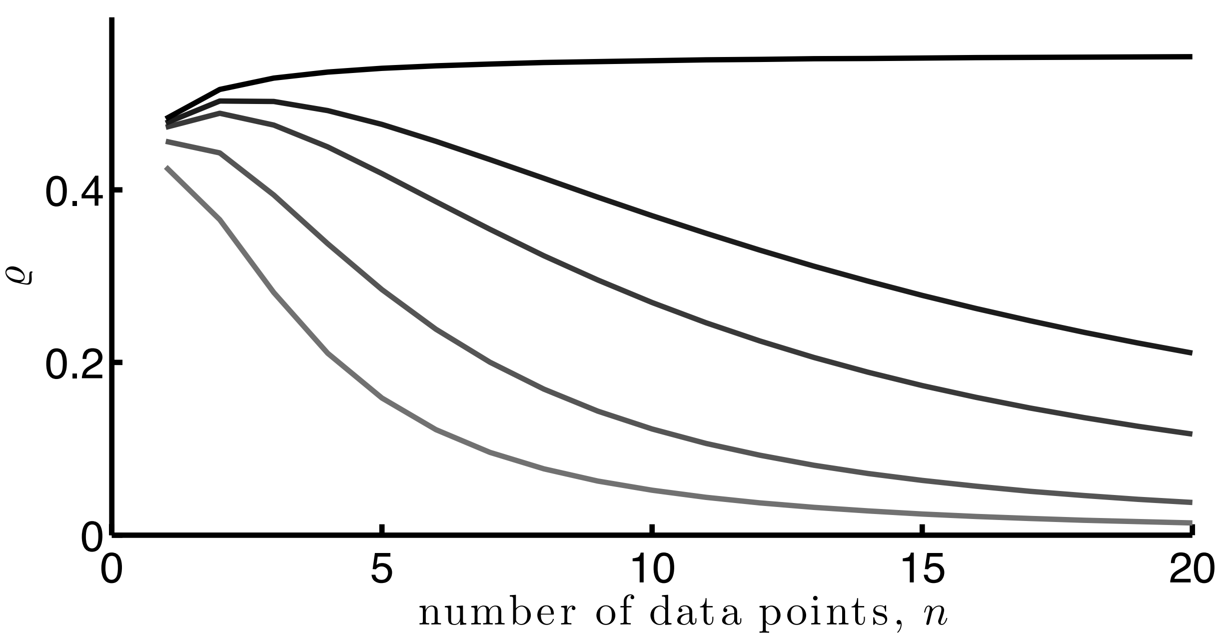

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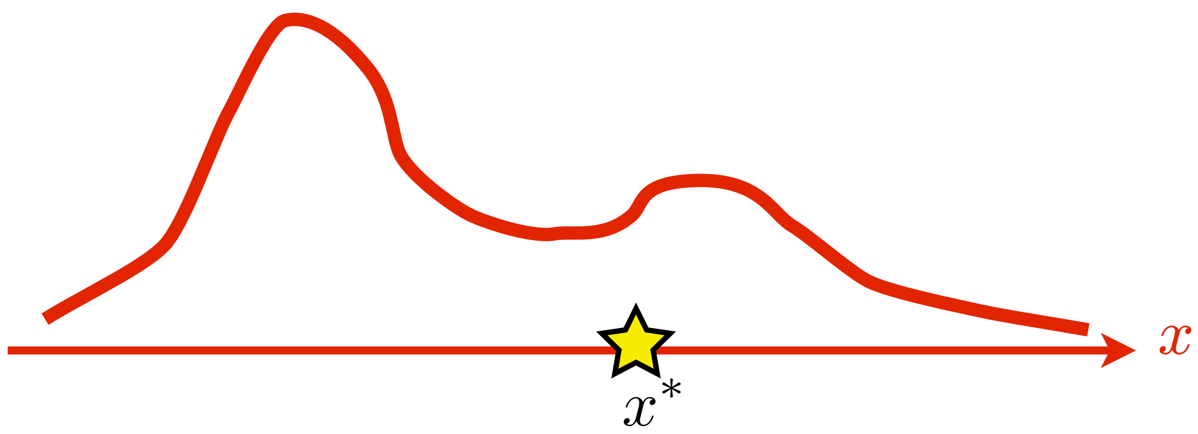
correlation can increase
(faster than error decreases)

large noise
correlation decreases

SAMPLING-BASED REPRESENTATION

Lengyel et al, arXiv 2015



assumption:

independent samples from the posterior

$$x_i \overset{\text{i.i.d.}}{\sim} \mathrm{P}(x|\mathcal{D})$$

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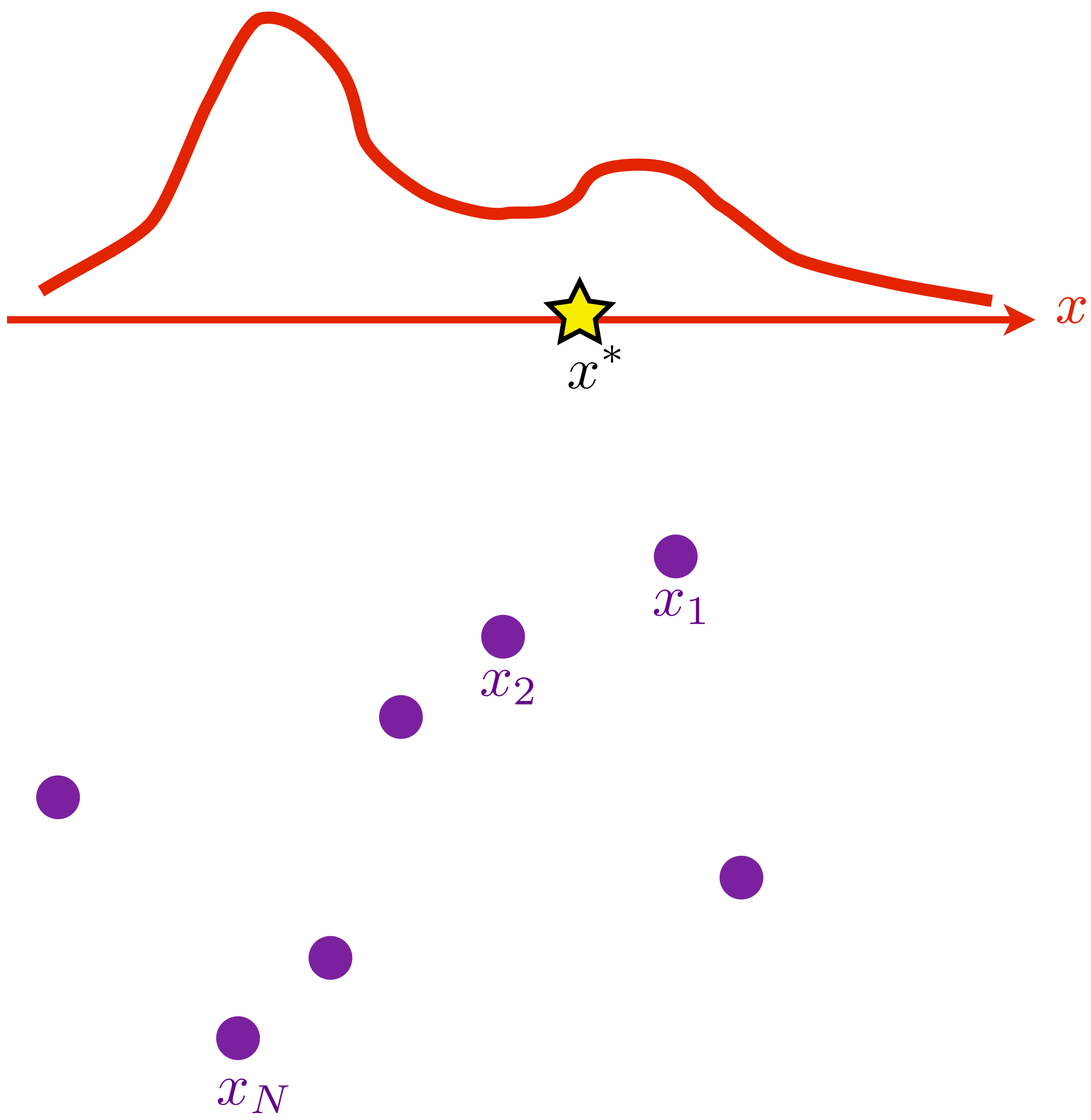
Lengyel et al, arXiv 2015



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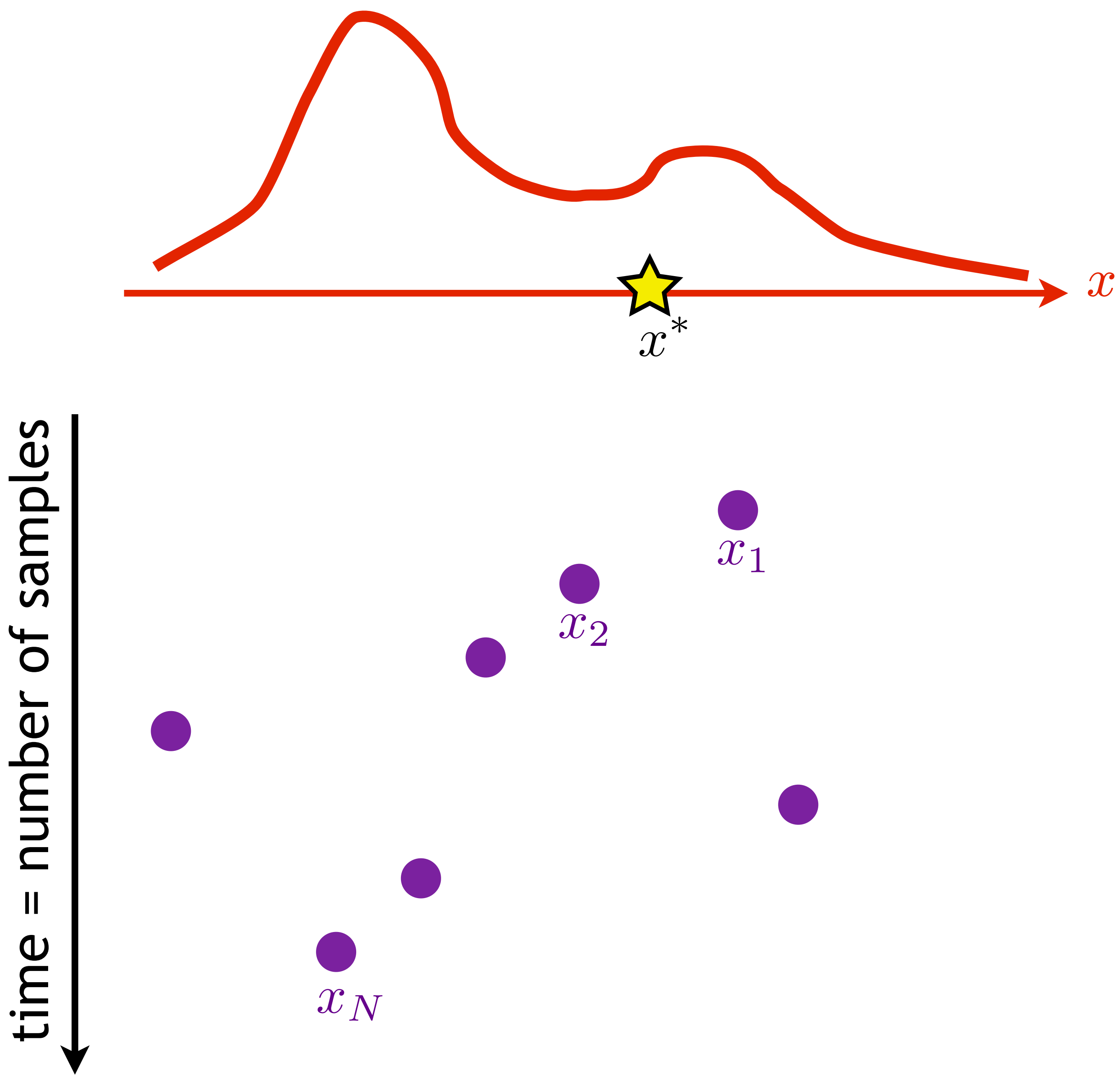

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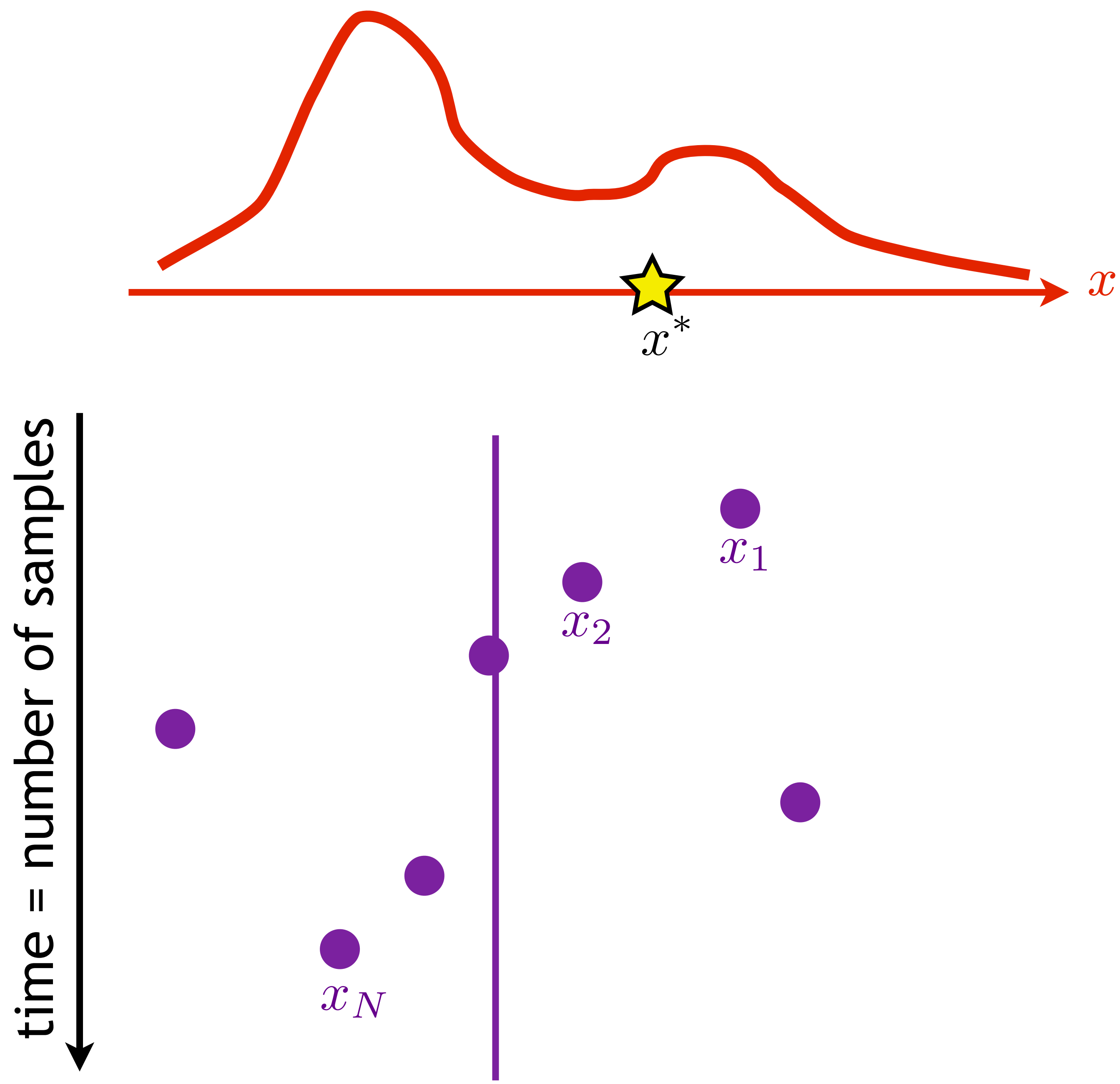

$\hat{\mu}_{\mathrm{x}}$: sample mean →

$\hat{\sigma}^2_{\mathrm{x}}$: sample variance →

assumption:

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behavioral reports

stimulus estimate

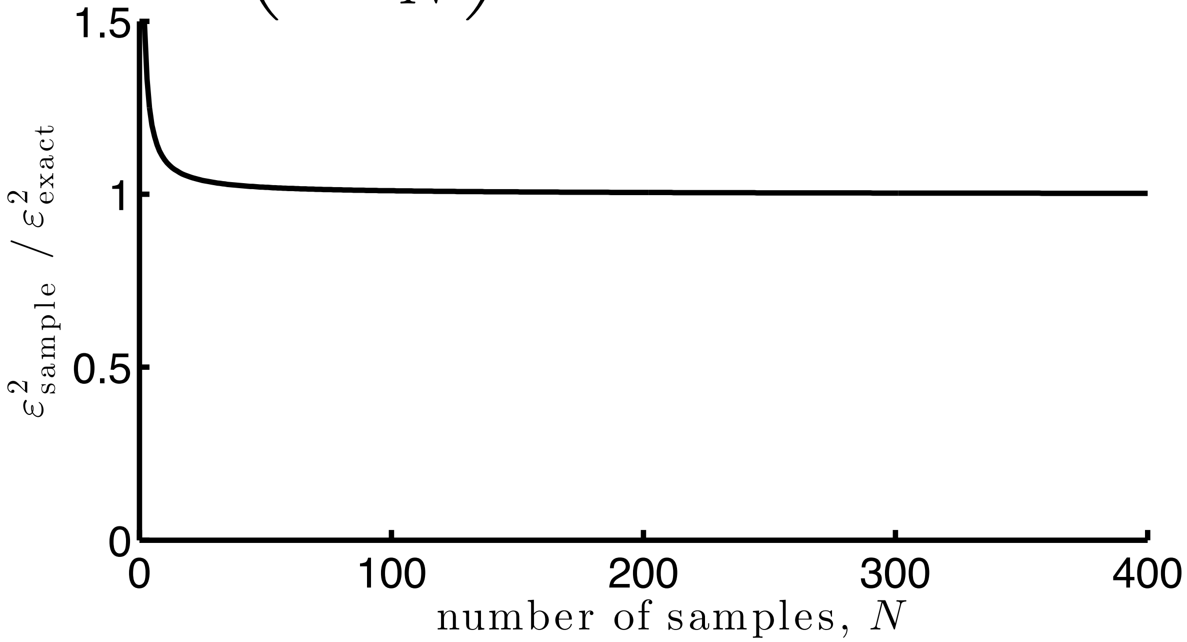
$$\hat{\mu}_{\mathrm{x}}$$

uncertainty

$$\left(1 + \frac{1}{N}\right) \hat{\sigma}^2_{\mathrm{x}}$$

mean squared error

$$\varepsilon^2 = \left(1 + \frac{1}{N}\right) \varsigma$$



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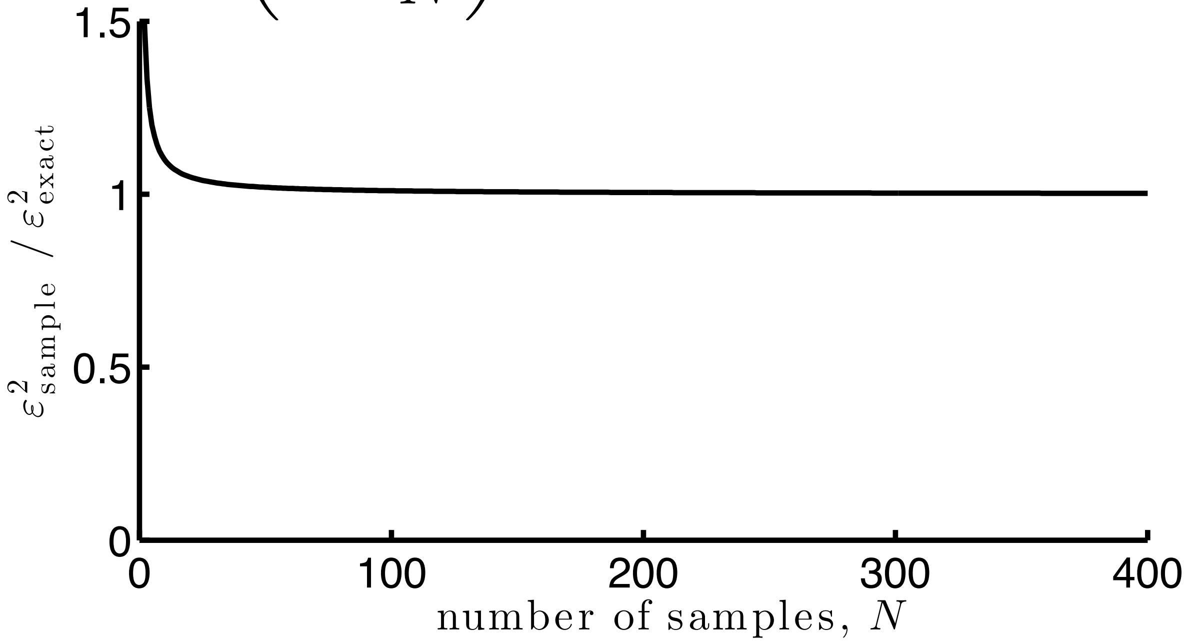
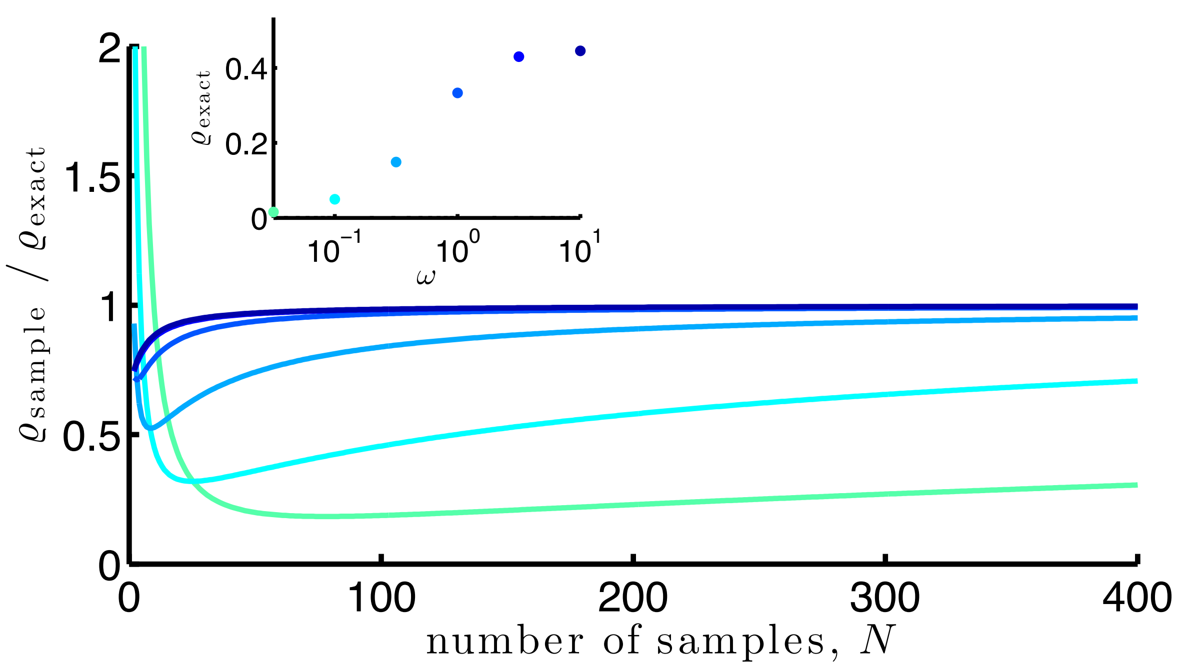


error-uncertainty correlation

$$\varrho \;=\; \frac{1 + \frac{1}{N} + \frac{\kappa_{\mathrm{x}}}{N^2}\,\left(\omega^{-2}+1\right)}{\sqrt{\left(\kappa_{\mathrm{x}} + 3 + (\kappa_{\mathrm{x}}+2)\,\omega^{-2} + \frac{1}{N}\left(6 + 4\,\omega^{-2}\right) + \frac{1}{N^2}\left(3+2\,\omega^{-2}\right) + \frac{\kappa_{\mathrm{x}}}{N^3}\,\left(\omega^{-2}+1\right)\right)}} \cdot \frac{1}{\sqrt{\left(1 + \left(\frac{\kappa_{\mathrm{x}}}{N} + \frac{2}{N-1}\right)\,\left(\omega^{-2}+1\right)\right)}}$$



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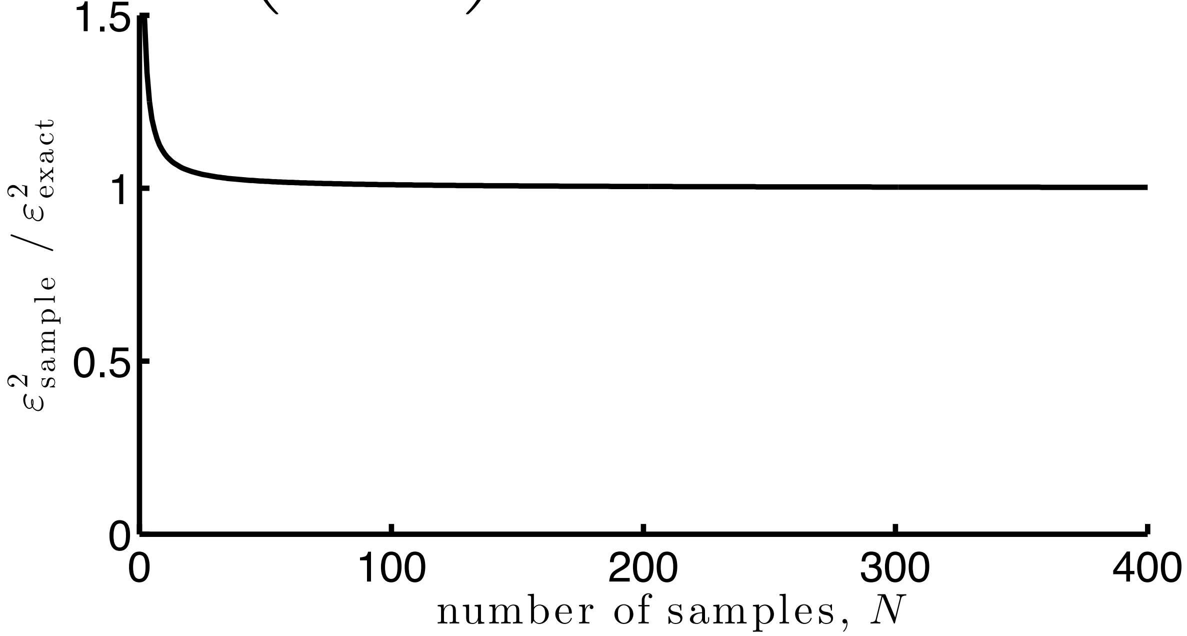

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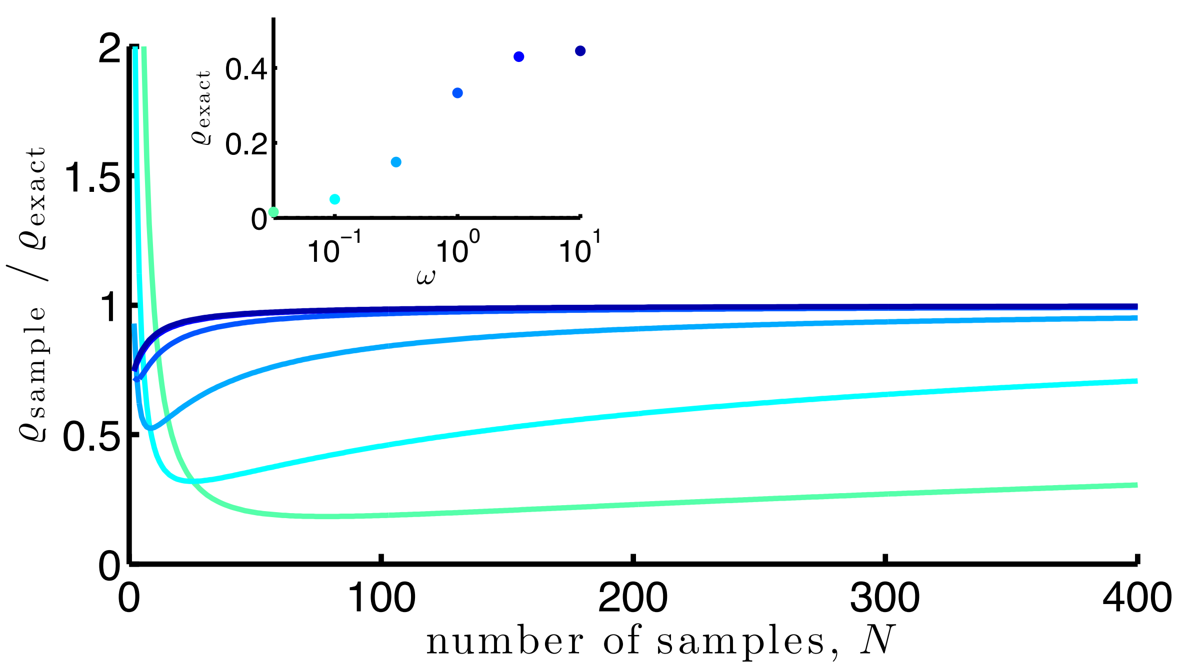

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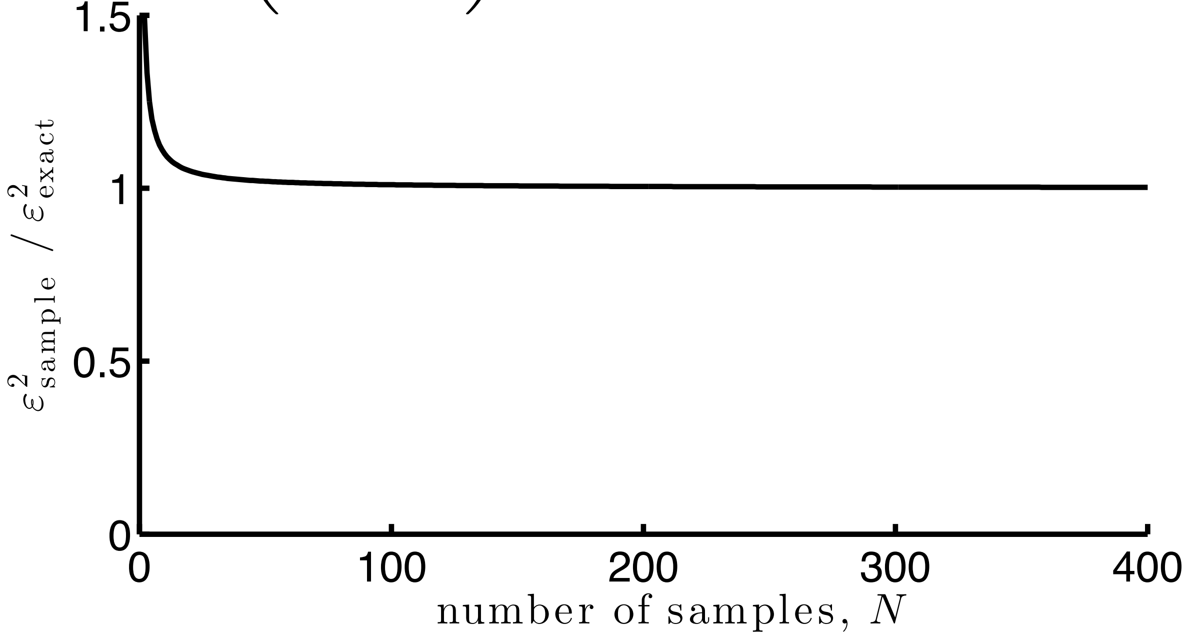

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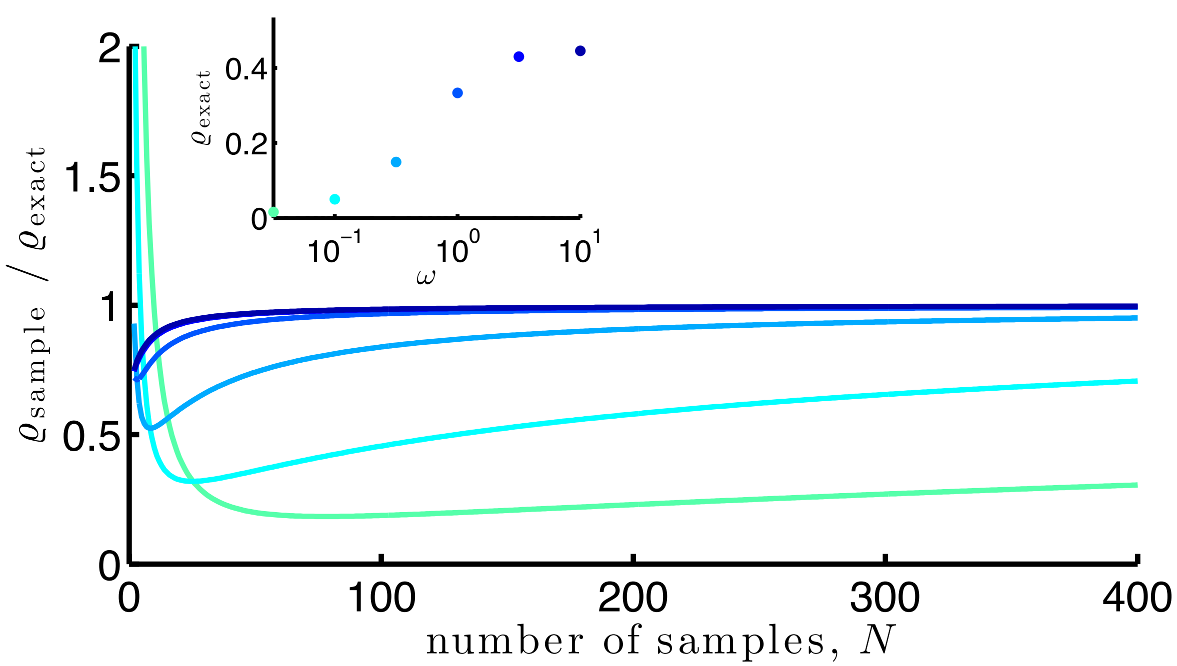

correlation first decreases transiently*
and then increases (slower than error)
(*when posterior kurtosis κ_{x}
is positive, and variability of trial
difficulty ω^2 is sufficiently large)

EFFECTS OF TIME

Lengyel et al, arXiv 2015

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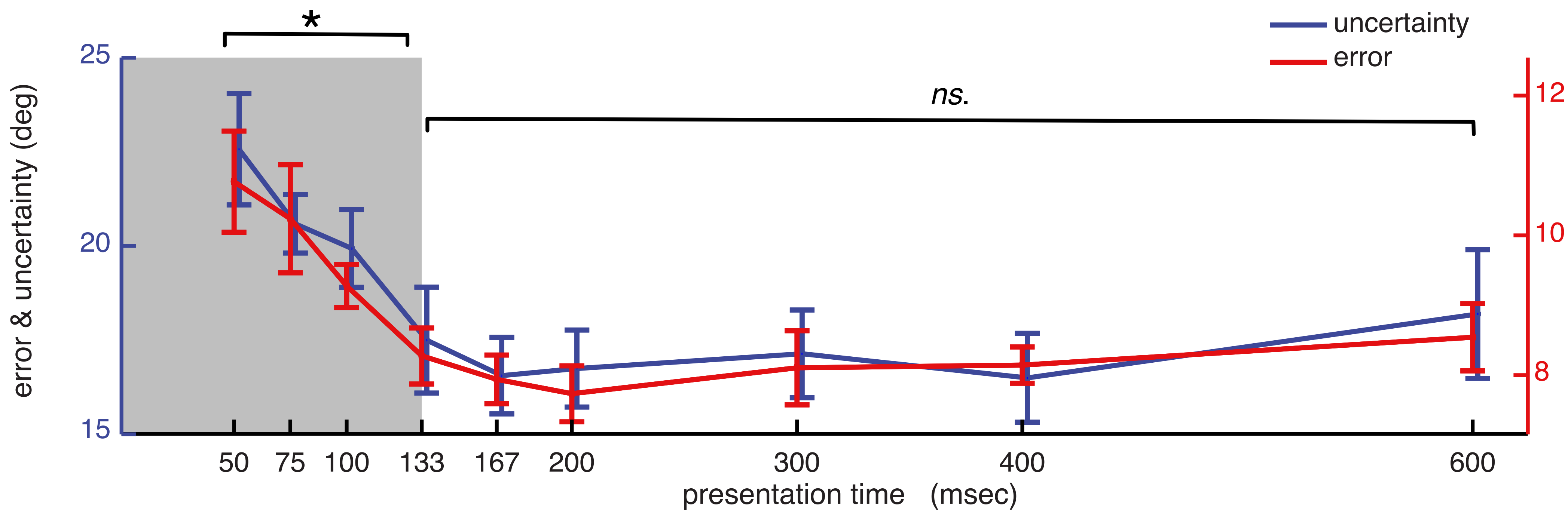
Lengyel et al, arXiv 2015

error and uncertainty



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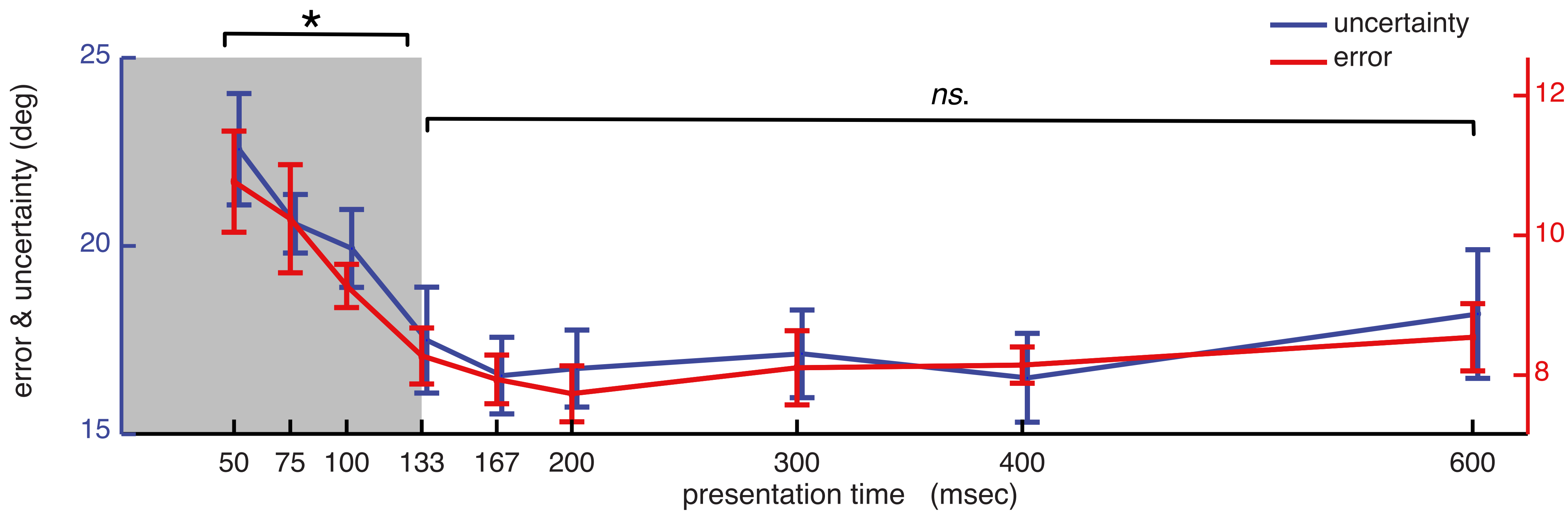

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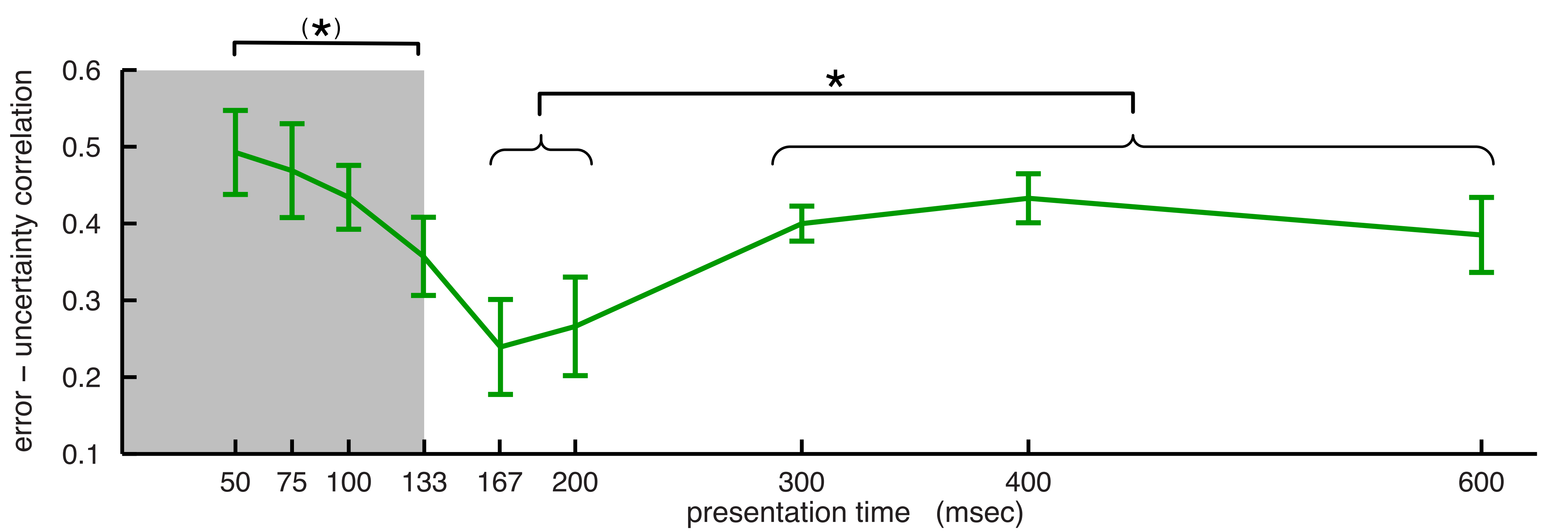

SUMMARY

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ACKNOWLEDGEMENTS

CEU

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wellcome trust

Investigator