Frequency Analysis
(Fourier, Spectral)

Experimental Study of Visual Perception

Question: How do we select the stimulus?

Gratings
1822 – J.B.J. Fourier, “Theorie analytique de la chaleur”.

- Fourier series – representation of periodic functions
- Fourier transform – representation of nonperiodic functions

Summary of Fourier Analysis

- Joseph Fourier (1768–1830) developed another useful tool for analyzing signals
  - Fourier analysis: A mathematical procedure by which any signal can be separated into component sine waves at different frequencies. Combining these component sine waves will reproduce the original signal
  - Sine wave:
    - 1. In hearing, a waveform for which variation as a function of time is a sine function. Also called a “pure tone”
    - 2. In vision, a pattern for which variation in a property, like brightness or color as a function of space, is a sine function.
A sine-wave is unequivocally defined by: (1) its amplitude (contrast); (2) its frequency; and (3) its phase.
Concept #1: To identify a sinusoidal function, you only three numbers are needed: Amplitude, phase, and frequency.

Frequency (or Fourier, or Spectral) Analysis

An example of sine waves: pure tones

- Sounds can be described as changes in pressure over time
- Tuning forks produce pure tones, which change pressure over time according to the sine function

An alternative representation (Fourier analysis)

- A signal with high peak amplitude
- A signal with a frequency of 12 Hz
An alternative representation (Fourier analysis)

An alternative representation (Fourier analysis)

An alternative representation (Fourier analysis)

An alternative representation (Fourier analysis)
Frequency (or Fourier, or Spectral) Analysis

Concept #1: To identify a sinusoidal function, you only need three numbers: Amplitude, phase, and frequency.

Concept #2: To graphically represent a sinusoidal function, you can just plot amplitude and phase as a function of frequency.

Concept #3: Every (practical) signal can be approximated by the sum of sinusoidal functions at different frequencies.
Frequency (or Fourier, or Spectral) Analysis

To get the full picture, you always need two graphs (amplitude and phase), but also amplitude alone is very informative.

1 cup oil
2 cup sugar
3 cup flour
1 tsp salt
1 tsp nutmeg
1 tsp cinnamon powder
1 tsp baking powder
3 eggs
1 tsp vanilla extract
1 cup chopped nuts
1 tsp cloves
1 cup baking soda
4 cups chopped apples
1 cup oil
1 cup sugar
3 cups flour
1 tsp salt
1 tsp nutmeg
1 tsp cinnamon powder
1 tsp baking powder
3 eggs
1 tsp vanilla extract
1 cup chopped nuts
1 tsp cloves
1 cup baking soda
4 cups chopped apples
Concept # 1: To identify a sinusoidal function, you only three numbers are needed: Amplitude, phase, and frequency.

Concept # 2: To graphically represent a sinusoidal function, you can just plot amplitude and phase as a function of frequency.

Concept # 3: Every (practical) signal can be approximated by the sum of sinusoidal functions at different frequencies.

Concept # 4: Every signal can be represented by two graphs showing the amplitudes and phases of the composing sinusoidal functions.
**Spatial frequency:** The number of cycles of a grating per unit of visual angle (usually specified in degrees)

(a)  (b)  (c)

Another way to think of spatial frequency is as the number of times a pattern repeats per unit area: (a) has a low spatial frequency, (b) has a medium spatial frequency, and (c) has a high spatial frequency.

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**Gratings provide an alphabet to build images**

Building complex patterns by adding gratings:

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**Amplitude = Contrast. Effects of changing contrast and frequency:** The top two gratings have same frequency and different contrast. The top and bottom grating differ in frequency but not in contrast.
Visibility of a pattern as a function of spatial frequency and contrast

<table>
<thead>
<tr>
<th>Spatial frequency (cycles/degree)</th>
<th>Contrast sensitivity</th>
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<tbody>
<tr>
<td>0.1</td>
<td>1000</td>
</tr>
<tr>
<td>1.0</td>
<td>100</td>
</tr>
<tr>
<td>10.0</td>
<td>10</td>
</tr>
<tr>
<td>100.0</td>
<td>Invisible</td>
</tr>
</tbody>
</table>

- Normal
- High frequencies filtered out
- Low frequencies filtered out

Contrast sensitivity vs. Spatial frequency

Visibility of a pattern as a function of spatial frequency and contrast
“All is vanity” Charles Allan Gilbert (1873-1929)

Summary of Fourier Analysis

- Why sine waves?
  - Many stimuli can be broken down into a series of sine wave components using Fourier analysis
    - Any sound, including music and speech
    - Any complex image, including photographs, movies, objects, and scenes
    - Any movement, including head and limb movements
  - Also, our brains seem to analyze stimuli in terms of their sine wave components!
    - Vision
    - Audition

Livingstone (2000)
<table>
<thead>
<tr>
<th>Summary of Fourier Analysis</th>
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<tbody>
<tr>
<td>• Properties of sine waves</td>
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<tr>
<td>▪ Period or wavelength: The time or space required for one cycle of a repeating waveform</td>
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<td>▪ Phase:</td>
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<tr>
<td>1) In vision, the relative position of a grating</td>
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<td>2) In hearing, the relative timing of a sine wave</td>
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<tr>
<td>▪ Amplitude: The height of a sine wave, from peak to trough, indicating the amount of energy in the signal</td>
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<td>• Images can be described as changes in light and dark across space. In the case of sine waves, these would look like bars of light and dark—gratings</td>
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<td>• Imagining a 360° circle around your head, your visual field is about 170° wide. Your thumbnail at arm’s length is about 1°. This is called “visual angle”</td>
</tr>
<tr>
<td>▪ Spatial frequency: The number of cycles of a grating per unit of visual angle</td>
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<tr>
<td>▪ Cycles per degree: The number of pairs of dark and bright bars per degree of visual angle</td>
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